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**MATHEMATICS**

# On the Possibility of Using the Method of Sign-Perturbed Sums for the Processing of Dynamic Test Data

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**Abstract**—At the present time, the methods for the measurement and prediction of the dynamic strength of materials are complicated and unstandardized. An experimental data processing method based on the incubation time criterion is considered. Only a finite number of measurements containing random errors and limited statistical information are usually available in practice, since dynamic tests are laborious, and every individual test requires a lot of time. This strongly restricts the number of applicable data processing methods unless we are satisfied with approximate and heuristic solutions. The method of sign-perturbed sums (SPS) is used for the estimation of finite-sample confidence regions with a specified confidence probability under the assumption of noise symmetries. It is shown that several experimental points are sufficient to determine the strength parameter with an accuracy acceptable for engineering calculations. The applicability of the proposed method is demonstrated in the processing of a number of experiments on the dynamic fracture of rocks.

**Keywords:** sign-perturbed sums, dynamic fracture, incubation time.

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## 1. INTRODUCTION

The measurement of the strength and rheological parameters of materials is one of the most important engineering problems. Such parameters as the static strength $\sigma_c$ or Young's modulus can be determined by existing standard methods because the values of these parameters can be measured directly. These measurement methods estimate a certain average required parameter value with a certain accuracy, e.g., $\sigma_c \in [\sigma_{c-}; \sigma_c^+]$. Moreover, the possibility of direct measurement implies that the physical meaning of a measured parameter has been defined and is well known, so all researchers agree on the obtained results.

A more complicated situation is observed in dynamics, where the strength of a material cannot be characterized by only the dynamic stress parameter. It is reasonable to call the amplitude of a loading pulse leading to the destruction of a material the "dynamic" strength $\sigma_d$. In the case where the load linearly grows, the "dynamic" strength $\sigma_d$ will always coincide with the fracture stress $\sigma_*$. Test results show that specimens under intense loading can momentarily endure a load which appreciably exceeds their static strength $\sigma_c$, and the fracture stress $\sigma_*$ depends on the loading rate and the destructing pulse shape [1–8]. Such rate–strength dependences are often interpreted in some theories as a property of materials. The main difficulty encountered in such an approach is the feasibility of an unlimited number of such strength curves owing to a strong effect of the mode and rate of impact loading on $\sigma_d$. Moreover, such an approach to material strength problems is incorrect in principle when the destruction of a material is provoked by threshold pulses. In this case, the fracture stress $\sigma_*$ may be much lower than the static strength. Such a phenomenon is called the fracture delay effect.

The structural-temporal approach [9, 10] based on the fracture incubation time is used for the prediction of dynamic fracture conditions. The main idea of this criterion is that fracture is not instantaneous, and each transient process has a certain characteristic fracture time $\tau$, i.e., the incubation time, which is a constant characterizing the strength of a material under high-rate dynamic loadings. The introduction of only one additional strength parameter, i.e., the incubation time $\tau$, provides the possibility to predict the fracture stress value and calculate the rate–strength dependences for all types of impact loads. This structural-temporal approach has been successfully applied for the solution of many problems of estimating the dynamic strength of different materials and continuous media, e.g., in such processes as the dynamic fracture of rocks and concrete, the high-rate plastic strain of metals, and the acoustic ultrasonic cavitation of liquids [10–13].

Experiments with direct measurements of incubation time are unfeasible, and the incubation time can currently be determined only by an implicit algorithm. The simplest method is to select the value of $\tau$ providing a good agreement between a model curve and a set of experimental data. However, the best possible match between a model curve and the optimal incubation time $\tau$ corresponding to it can be found by any method. The natural conditions of the least-squares method (LSM) provide a strongly consistent point estimate [14], but this method gives the only certain incubation time value without any error estimates in the case of observation variability. The parameter estimation error in the least-squares method is asymptotically normal, and this property can be used for the construction of approximate confidence regions. However, these regions are based on the central-limit theorem and, consequently, are ensured only asymptotically, as the number of experimental points tends to infinity. In this situation, standard algorithms of estimation usually use the continuous data disturbance condition. However, it is difficult to provide this condition in the considered problem, because dynamic tests are very complicated and laborious, so there is usually no great number of points for experimental data processing. This often leads to the degeneracy of experimental data and the appearance of complicated identification problems. In this context, the application of the classic system of identification theory with a finite number of experimental points leads only to heuristic confidence regions, which have no strict theoretical guarantees.

An alternative approach to identification for the estimation of a parameter range by the method of guaranteed sets does not use any statistical noise properties, but admits certain known upper boundaries for undefined system parameters as an alternative. The objective of this approach is usually to calculate certain upper or lower estimates on a set of data compatible with these parameters. The convergence of an obtained set of estimates to a true unknown vector parameter cannot be attained without any substantial additional assumptions (see, e.g., [15, 16]). In the context of the considered problem, the use of robust estimates of minimax methods gives a conservative result in the form of a very broad range for the incubation time $\tau$ with a great error.

In view of the aforesaid, it is necessary to use an alternative approach which will provide the construction of confidence regions with a specified degree of reliability around an LSM estimate under weak statistical hypotheses. Among such methods is the new sign-perturbed sums (SPS) algorithm [17] providing nonasymptotic confidence regions for an unknown multidimensional parameter in the linear regression model for a specified small sample. In this work, it is proposed to apply the SPS algorithm for the estimation of the incubation time $\tau$, which is a nonlinear model parameter describing the rate–strength dependence within the structural-temporal approach. This method is quite suitable for this problem, as it provides the confidence region of $\tau$ with an accuracy acceptable for engineering calculations. To determine the average incubation time with an error $\delta = 20–35\%$, it is sufficient to have approximately ten experimental points, and this guarantees a precise finite-sample confidence probability even in the absence of information on certain noise probability distributions. The main assumptions about noise conditions consist only in that they are independent and have symmetric distributions around zero, but their distributions may vary at every time point. The applicability of the new algorithm to the incubation time approach was demonstrated in several experiments on the dynamic fracture of different types of rocks [18]. This study is an extended version of the work [19], where the results of the processing of experiments on dynamic fracture of concrete are given.

## 2. INCUBATION TIME APPROACH

The structural-temporal approach has the following main properties. The incubation time criterion in the general form is

$$\frac{1}{\tau}\int_{t-\tau}^{t}\left(\frac{\sigma(t')}{\sigma_c}\right)^{\alpha} dt' \leq 1, \tag{1}$$

where $\sigma(t')$ is the loading stress, $\tau$ is the fracture incubation time, $t$ is the time moment at which it is checked whether there will be a fracture or not, and $\alpha$ is a dimensionless parameter ($\alpha = 1$ for most brittle materials). According to Eq. (1), fracture does not occur if the left-hand part of this criterion is less than unity. The fracture time $t_*$ corresponds to the time moment $t$ at which equality is attained for the first time.

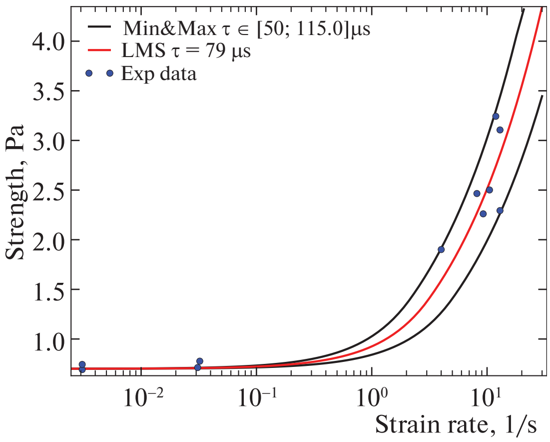


**Fig. 1.** Dynamic fracture of granite: experimental points (blue) and theoretical curves plotted using the fracture incubation time criterion by the least-squares method (red) and the minimax robust method (black).

Most of the experimental schedules for dynamic tests implement a linear increase in stresses until the moment of fracture. Then the load depends on the strain rate $\dot{\epsilon}$ and the elastic modulus $k$ as follows:

$$\sigma(t) = h(t)k\dot{\epsilon}t, \tag{2}$$

where $h(t)$ is the Heaviside step function. The substitution of Eq. (2) into criterion (1) leads to the following equation for the fracture time $t_*$:

$$h(t_*)\left(\frac{t_*}{\tau}\right)^{\alpha+1} - h(t_* - \tau)\left(\frac{t_*}{\tau} - 1\right)^{\alpha+1} = s, \tag{3}$$

where $s = (\alpha + 1)(\sigma_c/(k\dot{\epsilon}\tau))^{\alpha}$ is the dimensionless parameter depending on the loading rate. The fracture time $t_*$ cannot be negative, so $h(t_*) = 1$ and, consequently, two cases where $t_* > \tau$ or $t_* \le \tau$ must be considered. From Eq. (3), it follows that $s = 1$ at $t_* = \tau$, so the resulting equation for determining the fracture time moment $t_*$ takes the form

$$\begin{cases} (t_*/\tau)^{\alpha+1} = s, & s < 1, \\ (t_*/\tau)^{\alpha+1} - (t_*/\tau - 1)^{\alpha+1} = s, & s \ge 1. \end{cases} \tag{4}$$

Hence, the incubation time approach enables the prediction of rate dependences for the dynamic fracture threshold $\sigma_*(\dot{\epsilon}) = k\dot{\epsilon}t_*$. The parameter $\alpha = 1$ in most cases of calculations on the dynamic strength of brittle materials, such as rocks. The roots of Eq. (4) can be further expressed in an explicit form depending on the fracture mode and critical stress as

$$\sigma_*(\dot{\epsilon}) = \varphi(\tau, \dot{\epsilon}) = \begin{cases} \sigma_c + \frac{\tau}{2}k\dot{\epsilon}, & \dot{\epsilon} \le \frac{2\sigma_c}{k\tau}, \\ \sqrt{2\sigma_c\tau k\dot{\epsilon}}, & \dot{\epsilon} > \frac{2\sigma_c}{k\tau}. \end{cases} \tag{5}$$

The application of the incubation time criterion is exemplified in experiments on the impact fracture of granite and tuff. The theoretical curve in comparison with the experimental points [18] is shown in Fig. 1. The time $\tau = 79$ μs (solid line) was calculated by the least-squares method and, as mentioned above, this result gives no information about the error in the calculated value.

Another technique is the minimax method estimating the boundaries of the range from “terminal” experimental points, which results in a very broad range for the incubation time $\tau \in [50; 115]$ μs and also gives no information about whether new experimental points will lie within the found range. Hence, it is necessary to use a different data processing method which will give not only a narrower range of possible $\tau$ but also information about the degree of reliability for this range.

## 3. PROBLEM FORMULATION

The control parameter selected in the considered dynamic tests may be the strain rate $\dot{\epsilon}$, which corresponds to the observation $\sigma_*$ representing the fracture stress. In this case, experimental data satisfy the following model of $N$ noisy observations:

$$\sigma_{*i} = \varphi(\tau, \dot{\epsilon}_i) + v_i, \quad i = 1, 2, \ldots, N, \tag{6}$$

where $v_i$ are independent random noises (error) with a symmetric distribution. If the rate–strength dependence obeys the principles of the structural-temporal approach, the function $\varphi$ will be expressed as

$$\varphi(\tau, \dot{\epsilon}_i) = k\dot{\epsilon}t_*(\tau), \tag{7}$$

where $t_*$ is the fracture time predicted with the incubation time criterion by solving Eq. (4).

Equation (7) provides the calculation of the fracture stress at different $\tau$. Then the least-squares method can be used to determine the optimal incubation time, at which a minimum is attained by the following sum:

$$\sum_{i=1}^{N} (\varphi(\tau, \dot{\epsilon}_i) - \sigma_{*i})^2 \to \min_{\tau}. \tag{8}$$

However, we cannot obtain a sufficiently good confidence region for an unknown $\tau$ at a small number of tests $N$ without substantial constraints on random noises $v_i$.

### *Main Problem*

Our objective is to construct the confidence regions for the unknown parameter $\tau$ with a specified confidence probability for a finite and, more likely, small number of experimental points. These confidence regions must be determined for observed $\{\sigma_{*i}\}_{i=1}^{N}$ depending on the specified control factors $\{\dot{\epsilon}_i\}_{i=1}^{N}$, which can also be selected. The constructed regions have quasi-free distributions, as we make only two quite substantiated hypotheses about the distribution of random noises: they are independent random values, and their distribution has the property of symmetry. The formulation of such weak constraints is very important, as the knowledge about the noise distribution is very limited in practice. Moreover, the constructed confidence regions must contain a LSM estimate.

## 4. CALCULATION OF CONFIDENCE REGIONS BY THE SPS PROCEDURE

For a finite number of observations, it is possible to use the following procedure similar to the SPS procedure from [17]. According to the least-squares method, functional (8) will attain a minimum at an optimal incubation time, which is the solution of the equation

$$H_0(\tau) = \sum_{i=1}^{N} (\sigma_{*i} - \varphi(\tau, \dot{\epsilon}_i)) \frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau} = 0, \tag{9}$$

where

$$\frac{d\varphi(\tau, \dot{\epsilon})}{d\tau} = \begin{cases} \frac{1}{2}k\dot{\epsilon}, & \dot{\epsilon} \le \frac{2\sigma_c}{k\tau}, \\ \frac{1}{\sqrt{2\tau}}\sqrt{\sigma_c k\dot{\epsilon}}, & \dot{\epsilon} > \frac{2\sigma_c}{k\tau}. \end{cases} \tag{10}$$

If the prior statistical knowledge about noise is minimal, the information about the data is used. The main assumption is noise symmetry. For certain $M > 0$, $N(M - 1)$ random Bernoulli values $\beta_{ij} = \pm 1$ are generated with a probability of 1/2, and $M - 1$ sign-perturbed sums are introduced in the following form:

$$H_j(\tau) = \sum_{i=1}^{N} \beta_{ij}(\sigma_{*i} - \varphi(\tau, \dot{\epsilon}_i)) \frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau}, \quad j = 1, 2, \ldots, M - 1. \tag{11}$$

Let $\tau^*$ be the normal value of the incubation time $\tau$, i.e., a true parameter value, which would exist in nature without random noises. Then the parameters $H_0(\tau^*)$ and $H_j(\tau^*)$ have identical distributions owing to the above assumption about the independence and symmetry of random disturbances $v_i$, because we have $v_i = \sigma_{*i} - \varphi(\tau^*, \dot{\epsilon}_i)$ in this case according to Eq. (6). Hence, there are no grounds for which certain $|H_j(\tau^*)|$ must be higher or lower than its other value $|H_{j'}(\tau^*)|$, and the probability that certain $|H_j(\tau^*)|$ is the $m$th highest value from the ordered set $\{|H_j(\tau^*)|\}_{j=0}^{M-1}$ will be the same for all $j$ including $j = 0$ (the case without the multiplier $\beta_{ij}$) and equal to $1/M$.

*Algorithm*

(1) Taking into account the selected confidence probability value $p \in (0, 1)$, specify integers $M > q > 0$ such that $p = 1 - q/M$.

(2) Generate $N(M - 1)$ identically distributed independent random signs with probabilities $\{\beta_{ij}\}$ with $\mathrm{Prob}\{\beta_{ij} = 1\} = \mathrm{Prob}\{\beta_{ij} = -1\} = 1/2$ for $i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, M - 1$.

(3) Determine the range $\hat{\mathcal{T}} := \{\tau \in \mathbb{R} | \text{SPS\_indicator}(\tau) = 1\}$.

*Procedure SPS_indicator(τ)*

(1) Calculate the difference between the predicted and experimentally observed values of $\delta_i(\tau) = \sigma_{*i} - \varphi(t, \dot{\epsilon}_i)$, $i = 1, 2, \ldots, N$ at specified $\tau$.

(2) Calculate

$$H_0(\tau) = \sum_{i=1}^{N} \delta_i(\tau) \frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau}, \tag{12}$$

$$H_j(\tau) = \sum_{i=1}^{N} \beta_{ij} \delta_i(\tau) \frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau}, \quad j = 1, 2, \ldots, M - 1. \tag{13}$$

(3) Sort the set of scalars $|H_j(\tau)|$ in ascending order from lowest to highest.

(4) Calculate the range $\mathcal{R}(\tau)$ for $|H_0(\tau)|$ in the ordered set $\{|H_j(\tau)|\}_{j=0}^{M-1}$, where $\mathcal{R}(\tau) = 1$ if $|H_0(\tau)|$ is the lowest value, $\mathcal{R}(\tau) = 2$ if $|H_0(\tau)|$ is the second lowest value, and so on.

(5) Set the procedure SPS_indicator(τ) to 1 for $\mathcal{R}(\tau) \le M - q$ or 0, otherwise.

Let us note that the incubation time $\hat{\tau}$ calculated by the least-squares method is a root of the equation $H_0(\hat{\tau}) = 0$ by definition, so this estimate will be incorporated into the confidence region determined by the SPS procedure, i.e., $\hat{\tau} \in \hat{\mathcal{T}}$. The probability that $\tau^*$ belongs to $\hat{\mathcal{T}}$ is proven in the following theorem.

**Theorem.** If noises are independent random values with the property of symmetry, $\mathrm{Prob}\{\tau^* \in \hat{\mathcal{T}}\} = 1 - q/M$, where $M$, $q$, and $\hat{\mathcal{T}}$ are found from steps 1 and 3 of the above described algorithm.

**Proof.** This theorem is proven in the same way as the corresponding theorem in [17]. The main difference consists in the nonlinearity of the function $\varphi(\cdot, \cdot)$ from Eq. (5), but it is one-dimensional, convex, and monotonically ascending and continuous with respect to the parameter $\tau$. Indeed, according to Eq. (10), the function derivative $d\varphi(\tau)/d\tau$ is positive for any $\tau > 0$, and the critical loading rate $\dot{\epsilon}_c = 2\sigma_c/k\tau$ continuously decreases with increasing $\tau$ in this case. This means that $\varphi(\tau_1, \dot{\epsilon}) > \varphi(\tau_2, \dot{\epsilon})$ for any $\dot{\epsilon}$ when $\tau_1 > \tau_2$. Hence, when $\tau \notin [\tau_{\min}; \tau_{\max}]$, i.e., $\tau$ does not belong to the range found by means of minimax estimation, curves $\varphi(\tau, \dot{\epsilon})$ will lie outside the min & max range (Fig. 1), i.e., above or below experimental points. Let us consider the case where $\tau < \tau_{\min}$ and all the predicted strengths are lower than their experimentally observed values, i.e., $\varphi(\tau, \dot{\epsilon}_i) < \sigma_{*i}$, $\forall i = 1, \ldots, N$. Then all the summands in $H_0(\tau)$ (Eq. (9)) will be positive, and the absolute value of this sum will exceed any sum $H_j(\tau)$ from Eq. (11), because some of its summands change their sign to negative, i.e., $|H_0(\tau)| > |H_j(\tau)|$, $\forall \tau < \tau_{\min}$, and $j = 1, \ldots, M - 1$. In this case, we have $\mathcal{R}(\tau) = M$, and the corresponding value of $\tau$ is excluded by the procedure SPS_indicator(τ). Similarly, $\tau > \tau_{\max}$ will also be excluded, because all the summands of $H_0(\tau)$ will be negative in this case in contrast to $H_j(\tau)$. Therefore, we obtain $|H_0(\tau)| > |H_j(\tau)|$ and $\mathcal{R}(\tau) = M$. Hence, the range $\hat{\mathcal{T}}$ determined by the SPS algorithm will be confined within the min & max range. Moreover, according to Lemmas 1–3 from the work [17], the probability that any $H_j(\tau)$ at $\forall \tau \in \hat{\mathcal{T}}$ occupies any certain position in the ordered set $\{|H_j(\tau)|\}_{j=0}^{M-1}$ is $1/M$ and, consequently, $\mathrm{Prob}(\mathcal{R}(\tau) \le M - q) = 1 - q/M$.

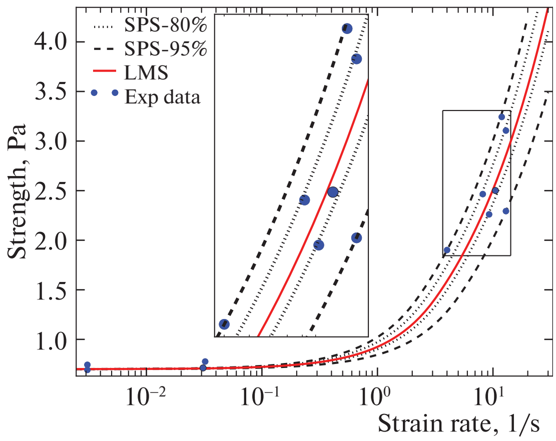


**Fig. 2.** Dynamic fracture of granite [18]: experimental points (blue) and theoretical curves plotted by the least-squares method (red) and the SPS procedure for $\tau_{0.8} \in [71; 96]$ μs (dotted) and $\tau_{0.95} \in [51; 112]$ μs (dashed).

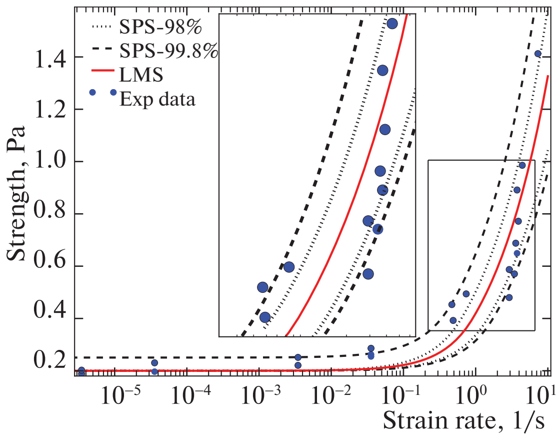


**Fig. 3.** Dynamic fracture of tuff [18]: experimental points (blue) and theoretical curves plotted by the least-squares method (red) and the SPS procedure for $\tau_{0.98} \in [605; 1400]$ μs (dotted) and $\tau_{0.998} \in [511; 2100]$ μs (dashed).

The above theorem should be immediately complemented by the following remarks:

**Remark 1.** The natural upper constraint for the number of sets generated for the random signs of $M$ at $N$ observations is $M < 2^N - 2$.

**Remark 2.** The lower boundary for the number of sets generated for the random signs of $M$ is determined by the degree of reliability for the calculated range $p$: $M > 1/(1 - p)$. However, the experience of calculations shows that it is sufficient to select $M \sim 20$–40 for the construction of confidence regions with a relatively high degree of reliability of 70–90%.

## 5. EXPERIMENTS

The results of applying the SPS procedure for the processing of experimental data on the dynamic fracture of different types of rocks are shown in Figs. 2 and 3. For the dynamic fracture of granite (see Fig. 2), two values for the confidence probabilities of 80 and 95% were attained at the following SPS procedure parameters: $M = 20$, $q = 4$ and $M = 20$, $q = 1$. This gives two regions for the fracture incubation time: $\tau_{0.8} \in [71; 96]$ μs and $\tau_{0.95} \in [51; 112]$ μs; the least-squares method provides $\tau = 79$ μs.

Hence, the SPS procedure provides the calculation of the incubation time at a relatively low error of about 22% with a rather high confidence probability of 80%.

For the dynamic fracture of tuff (see Fig. 3), we have a similar situation for the following parameters of the SPS procedure: $M = 50$, $q = 1$ for 98% and $M = 500$, $q = 1$ for 99.8%. In this case, the 98% error of the confidence region $\tau_{0.98} \in [605; 1400]$ μs is only about 40% with respect to the LSM estimate $\tau = 977$ μs. The second confidence region $\tau_{0.998} \in [511; 2100]$ μs with an error of 99.8% according to the theorem tends to the region determined by the minimax method.

It should be noted that a different number of experimental points was used for each material in the calculations, but this had no effect on the applicability of the SPS algorithm. This algorithm gives the incubation time value with a degree of accuracy of approximately 20–40% acceptable for engineering calculations.

## 6. CONCLUSIONS

The main problem in dynamic fracture mechanics is the absence of standard methods for the calculation and prediction of the ultimate characteristics of intense impacts for different materials and continuous media. The structural-temporal approach has proven to be a good tool for the solution of the mentioned problems, but no standard procedure for estimating the incubation time has been defined for it. The main difficulty consists in that the incubation time $\tau$ can be measured only in an indirect way, and standard measurement methods are inapplicable. The proposed application of the SPS procedure shows how the fracture incubation time can be calculated with allowance for a specified error $\delta$ at a limited number of experimental points. This algorithm also makes it possible to estimate the reliability of found regions for the parameter $\tau$. In the future, it is planned to make weaker assumptions about noise such as in [20–22] and extend the results obtained to the function $\varphi(\cdot, \cdot)$ with $\alpha \neq 1$ (see Eq. (4)).

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