

# Social Capital of Professors as a Factor of Increasing the Effectiveness of the University

Sergey Yu. Trapitsin

Institute of Economics and Management  
Herzen State Pedagogical University  
St. Petersburg, Russia  
trapitsin@gmail.com

Olga A. Granichina

Institute of Childhood  
Herzen State Pedagogical University  
St. Petersburg, Russia  
olga\_granichina@mail.ru

Oleg N. Granichin

Faculty of Mathematics and  
Mechanics, and Research Laboratory for Analysis and  
Modeling of Social Processes  
Saint Petersburg State University,  
St. Petersburg, Russia  
oleg\_granichin@mail.ru

**Abstract - Possible approaches to building the conceptual and mathematical models to the measurement of social capital of the University are discussed in this paper. We characterize the structure of social capital, determine the factors affecting the level of social capital, identify indicators of its estimation, formulate the problem of predict the effectiveness of the University through the organization of measurement by these indicators. The problems associated with the measurement of social capital are described. Discusses the choice of the mathematical model of the organization of estimation of social capital in the context of determining the confidence interval of the permissible values of the effectiveness of the University as a basis for making management decisions. Examines the complexity of the task of estimation of social capital at the University, posit that the decision these problems by standard methods is very difficult. For the solution of this task offers to use the method of sign-perturbed sums (SPS). This paper develops the previous ideas of the authors on the use of randomized algorithms in socio-humanitarian researches, in which the traditional assumptions about of stochastic character, independence and the symmetry of noise of the observations are often made unreasonably.**

**Keywords – evaluate of social capital and effectiveness of the University; the structure of social capital; conceptual and mathematical models of social capital; the method of sign-perturbed sums**

## I. INTRODUCTION

The analysis of foreign and Russian researches shows that the human factor is of crucial importance in improving the efficiency of universities, and the effectiveness of the University depends on the efforts and results of the work of professors, their competence, motivation, creative potential, innovation activity. As a result, the range of research objects in the University management is expanding, and the analyzed pressure groups increasingly include teams of professors and professional communities. Multiple researches have shown that constructive social relations, the using of resources of

professional networks, mutual trust and other parameters, which we are usually identified as social capital, increasingly determine the effectiveness of the University. Thus, the actual scientific and practical problem is the problem of analyzing the structure of social capital, its description (modeling), evaluation the degree of its impact on the indicators of individual (professors) and group (Departments, faculties, University) efficiency.

## II. MODEL FOR MEASURING SOCIAL CAPITAL

### A. Content model

Social capital is associated with the possibilities of "horizontal" (informal interpersonal) and "vertical" (formal, hierarchically organized) relations of professors. Its incorporation into the University management system can provide intensification of human capital (motivation, knowledge, experience, skills), as well as a synergistic effect by harmonizing between the individual goals of professors (groups of professors) and the goals of the University.

We assume that the characteristics of social capital in horizontal and vertical aspects are interrelated, which can be further confirmed by the growth of individual and group productivity depending on the total quantity of social resources that available to the agent.

Modeling the structure of social capital is complicated by methodological problems. In particular, it is a problem of definition of complex and ambivalent concept of "social capital" and its multidimensionality. There is no common, universally recognized concept of social capital in present time, it due to its dual nature. On the one hand, social capital is defined as a set of attributes of interacting individuals, and on the other hand – it is defined as a set of attributes of the environment in which individuals interact. However, social capital has several recognized characteristics: it is based on trust, shared norms and values in society, and the parameters of network communications. At the same time, modeling each of

these characteristics is a non-trivial research task, since the structure of social capital is formed in such different economic, political and cultural conditions that the comparability of data is rather surprising than demonstrate regularity. This complicates the task of diagnostics and analysis of social capital, leads to the need for multiple measurements and to the complexity of creation reliable predictions of the impact of social capital on the effectiveness of the University because of the conditions are non-repeatability.

To measure the first component of social capital (characteristics of network agents), five groups of indicators are traditionally used:

- a) common level of personal trust
- b) level of institutional trust
- c) characteristics of group identity (belonging to the department, faculty, University, academic community)
- d) perceived social capital of the environment
- e) perceived social capital of institutions.

The modification of semantic differential can be used as a measuring method. It allows reconstructing the collective structure of representations by the main components of social capital: trust, solidarity, tolerance and identity. Scaling results are processed in a standard way: factor analysis of the results is carried out, the name of the identified factors is given, factor weights of scaling objects are calculated, scaling objects are located in the space of the selected factors in accordance with their weights.

Measurement of the second component of social capital (characteristics of the network) includes analysis of internal and external relations: focus (internal, external), quantity and quality of connections, their intensity and configuration, network resources available due to these connections, etc.

Another problem of social capital research is the organization of information collection and the ability to adequately measure the selected indicators. The first type of problem here is that the situations simulated by the surveys are hypothetical and do not reproduce the real experience of the agent. The second type of problem occurs because each individual has a subjective assessment of himself, and often he does not answer on behalf of who he really is, but on behalf of who he would like to be. This can be avoided if we involve the development of correct questionnaires professionals, but this leads to an increase in research costs, so the task of minimizing the number of measurements becomes very important. The third problem is caused by the high cost of existing methods of measuring social capital. In some cases, respondents have no interest in providing reliable information, they perceive the survey not as a source of research data, but as a way to earn income. Unfortunately, this distortion is difficult to overcome, and the researcher can never be sure of the honesty of the respondent's answers (Carpenter, 2002), and it also requires the development of ways to minimize the cost of measuring social capital.

Trust is the most studied area of all the components of social capital, so in the subsequent discussion we will

deliberately simplify the task of assessing social capital to measure this characteristic solely. This will not have a fundamental impact on the understanding of the general research methodology associated with the development and use of the proposed mathematical model.

Methods of measuring trust can be divided into two groups: the first group is the opinions of people regarding the level of trust (attitudinal measure of trust); the second group allows you to measure trust through the observation of the behavior of people in situations that could characterize their level of trust (behavioral measure of trust). Based on this categorization, the first group of methods is a survey of individuals, and the second group of methods is conducting experiments (games) (Capra, etc., 2008).

In most cases the measurement of trust level is based on the model of World Values Survey (conducted since 1981), the purpose of which is to study values and beliefs and their changes in time. In different studies about the trust the authors use this model, sometimes adding additional questions in survey. However, some researchers do not consider an objective and adequate assessment of trust obtained by measuring the opinions of individuals about trust or distrust (Glaeser and al., 2000), they offer to study not only the perception of trust, but also the behavior (real actions) characterizing the level of trust/distrust in a multi-agent network, as well as the assessment of the ability to inspire trust. This problem can be solved partially by the using of game theory when social capital is presented as a propensity to make cooperative decisions, even if it is not a Nash equilibrium (Paldam, 2000). An example is the classic «prisoner dilemma» game, where the measurement of social capital is the frequency with which players play a cooperative solution, moreover players can be completely unfamiliar with each other and the game can be played only once.

#### *B. Mathematical model*

An adequate solution to any problem is possible with a precise statement of the task, but connections and relationships in the real world are so complex and diverse that it is almost impossible to describe difficult-to-formalize processes and phenomena mathematically strictly. A typical approach in the theory is the choice of the mathematical model, adequate to the real processes, and the inclusion in it of various types of noise, determined, on the one hand, the roughness of the mathematical model and, on the other hand, characterize the uncontrolled external perturbations on the object or system. From this point of view a large number of applied problems of the management of the effectiveness of the University can be interpreted as the restoration of an unknown dependence on empirical data. To solve the practical problems of efficiency control optimization, the most adequate is the use of randomized algorithms that allow in many cases to get off the negative impact of systematic errors (Granichin O.N., 2012). In the case of a sufficiently large number of observations of the object with dynamic characteristics, it is possible to apply the traditional approach based on the least-squares method (LSM). However, there are situations where it is not possible to make great number of observations, or where the data are difficult to measure, or where the dynamics of changes of the object are too high. This is certainly the case of measuring social capital.

In these cases, a good result can be obtained by using the sign-perturbed sums (SPS) method, which provide the construction of confidence regions with a specified degree of reliability for an unknown multidimensional parameter for a small number of observations. A significant limitation of the applicability of this method can be eliminated by randomization of the input effects in the modified sign-perturbed sums method (Granichin O.N., Senov A.A., Amelin K.S., Amelina N.O., 2014).

The main idea of the method is described below.

Let  $f(u, \vartheta)$  be known function of two vector arguments:  $u \in \mathbb{R}^k$  и  $\vartheta \in \Theta \subseteq \mathbb{R}^d$ , i.e.  $f: \mathbb{R}^k \times \Theta \rightarrow \mathbb{R}$ , and it has a continuous Frechet derivative with respect to the second argument at all interior points of the set  $\Theta$ . Let function  $f$  is adequate model of some real phenomenon (the impact of social capital on the efficiency of the University) with a single unknown value of the vector parameter  $\vartheta = \vartheta^*$ . Now, instead of the task of calculating the impact assessment, we get the task of constructing a confidence region for an unknown parameter  $\vartheta^*$ . Note that in the studies of the relationship between the social capital and the effectiveness of the University, individual trust indices of the network agents within the study population (department, faculty, University) can be taken as input parameters, and the output parameters will be the values of the group efficiency of the multi-agent network (department, faculty, University).

Consider the model of observations:

$$y_t = f(u_t, \vartheta^*) + v_t, \quad t = 1, \dots, T \quad (1)$$

where  $y_t$  are observations (outputs),  $y_t \in \mathbb{R}$ ,

$v_t$  are random external noise,  $v_t \in \mathbb{R}$ ,

$u_t$  are known observation plan (inputs) that is given by the experimenter or selected somehow randomly from some set  $U \subset \mathbb{R}^k$ ,  $\vartheta^* \in \Theta \subseteq \mathbb{R}^d$ ,

$\vartheta^* \in \Theta \subseteq \mathbb{R}^d$  is a true value of the unknown vector parameter belonging to some given set  $\Theta \subseteq \mathbb{R}^d$ ,

$t$  is an experiment number,  $T$  is a total number of experiments.

It is required to construct a confidence set  $\widehat{\Theta}_T \subseteq \Theta$  such that  $P(\vartheta^* \in \widehat{\Theta}_T) \geq p$  for a given sequence of inputs  $u_1, u_2, \dots, u_T$  and observations  $y_1, y_2, \dots, y_T$  with user-defined confidence level  $p$  from interval  $[0, 1]$ .

#### Modified sign-perturbed sums method

The model of observations is produced more complex than the standard model in the SPS method.

Let  $\Delta_t, t = 1, 2, \dots, T$  be a sequence of independent, equally distributed random variables equal to  $\pm 1$  with the same probability of 1/2.

Suppose that the function  $f(\cdot, \cdot)$  is differentiable not only by the second, but also by the first argument, and for a fixed measurement plan  $u_t$  for each moment  $t = 1, 2, \dots, T$  except observations (1) made more observations

$y_t^+ = f(u_t^+, \vartheta^*) + v_t^+$  in the disturbed points of inputs  $u_t^+ = u_t + \Delta_t$ .

For three observed values  $y_t, y_t^+, \Delta_t$  at each moment define  $t = 1, 2, \dots, T$  a "new" dimension

$$\bar{y}_t := \Delta_t(y_t^+ - y_t) \quad (2)$$

For the sequence of measurements  $\{\bar{y}_t\}$  in according with (1) and (2) we have

$$\bar{y}_t = \bar{f}(u_t, \Delta_t, \vartheta^*) + \bar{v}_t \quad (3)$$

where  $\bar{f}(u_t, \Delta_t, \vartheta^*) = \Delta_t(f(u_t^+, \vartheta^*) - f(u_t, \vartheta^*))$  and

$\bar{v}_t = \Delta_t(v_t^+ - v_t)$ . According to the mean-value theorem, the function  $\bar{f}(u_t, \Delta_t, \vartheta^*)$  can be represented as  $\bar{f}(u_t, \Delta_t, \vartheta^*) = \Delta_t \nabla_u f(u_t', \vartheta^*) \Delta_t = \Delta_t^2 \nabla_u f(u_t', \vartheta^*) = \nabla_u f(u_t', \vartheta^*)$  (4)

where  $u_t'$  is some point between  $u_t$  and  $u_t^+$ .

For correct construction of confidence region by SPS method it is necessary to take the following assumptions:

1. The noise sequence  $\{v_t\}$  and  $\{\nabla_u f(u_t', \vartheta^*)\}$  independent from  $\{\Delta_t\}$ .

2. The matrix

$$R_T^+(\vartheta) = \frac{1}{T} \sum_{t=1}^T [\nabla_\vartheta f(u_t^+, \vartheta) \nabla_\vartheta f(u_t^+, \vartheta)^\top + \nabla_\vartheta f(u_t, \vartheta) \nabla_\vartheta f(u_t, \vartheta)^\top - \nabla_\vartheta f(u_t^+, \vartheta) \nabla_\vartheta f(u_t, \vartheta)^\top - \nabla_\vartheta f(u_t, \vartheta) \nabla_\vartheta f(u_t^+, \vartheta)^\top] > 0 \quad (5)$$

is nondegenerate for all  $\vartheta \in \Theta$ .

3. The gradient of the function  $\nabla_\vartheta f(u, \vartheta)$  is uniformly bounded and the function  $f(\cdot, \cdot)$  satisfies the Hölder condition

$$\|\bar{V}_j\| \leq M \|\vartheta^* - \vartheta\|^\alpha, \text{ where } M > 0, \alpha \leq 1,$$

$$\bar{V}_j = (\nabla_\vartheta f(u_j^+, \vartheta) - \nabla_\vartheta f(u_j, \vartheta))(f(u_j^+, \vartheta^*) - f(u_j^+, \vartheta) - (f(u_j, \vartheta^*) - f(u_j, \vartheta))), \quad j = 1, \dots, T.$$

4. For arbitrary  $j, k = 1, 2, \dots, T$  for sufficiently distant  $\vartheta$  from  $\vartheta^*$  it is valid

$$[\bar{V}_j]^\top T(\bar{R}_T^+)^{-1}(9)\bar{V}_k \geq \mu \|\vartheta^k - \vartheta\|^\rho > 0 \text{ with some constants } \mu > 0 \text{ and } \rho > 1: \rho > \alpha.$$

The symmetric matrix  $\bar{R}_T^+(\vartheta)$  is that  $(\bar{R}_T^+(\vartheta))^\top \bar{R}_T^+(\vartheta) = \bar{R}_T^+(\vartheta) \bar{R}_T^+(\vartheta)$ .

If assumptions 1-4 be valid, that

$$P(\vartheta^* \in \widehat{\Theta}_T) = p = 1 - q/M \text{ and set } \widehat{\Theta}_T \text{ be bounded.}$$

Procedure MSPS-indicator ( $\vartheta, p$ )

(1) Let  $\Delta_t, t = 1, 2, \dots, T$  be a sequence of independent, equally distributed random variables equal to  $\pm 1$  with the same probability of 1/2.

(2) Get the experimental data and evaluate  $\bar{y}_t$  by the (3).

Set integers  $q, M$  such that

$$M > q > 0 \text{ и } p = 1 - q/M.$$

(3) Generate  $(M-1)T$  i.i.d. random signs

$$\beta_{it} = \pm 1: P(\beta_{it} = 1) = P(\beta_{it} = -1) = 1/2$$

for  $t = 1, 2, \dots, T$  and  $i = 1, 2, \dots, M-1$ .

(4) For the given  $\vartheta$  compute the prediction error  $\delta_t(\vartheta) = \bar{y}_t - \bar{f}(u_t, \vartheta), t = 1, 2, \dots, T$ .

(5) Evaluate  $R_T(\vartheta)$  by the (5) and do the factorization.

Evaluate

$$H_0(\vartheta) := \bar{R}_T(\vartheta) - 1/2 \sum_{t=1}^T \nabla_\vartheta \bar{f}(u_t, \vartheta) \delta_t(\vartheta),$$

$$H_i(\vartheta) := \bar{R}_T(\vartheta) - 1/2 \sum_{t=1}^T \beta_{it} \nabla_\vartheta \bar{f}(u_t, \vartheta) \delta_t(\vartheta),$$

$$i = 1, 2, \dots, M-1.$$

(6) Compute the rank

$$\vartheta: \mathcal{R}(\vartheta) = \mathbf{R}(\|H_0(\vartheta)\|, \dots, \|H_{M-1}(\vartheta)\|).$$

(7) Return 1 if  $\mathcal{R}(\vartheta) \leq M - q$ , otherwise Return 0.

The position calculation

If the set of values  $G_1, \dots, G_{M-1}$  has not one that equal to the value  $G_0$ , then we call the position of the value  $G_0$  in this set rank and be  $\mathbf{R}(G_1, \dots, G_{M-1})$  the number of the value  $G_0$  in the ascending series  $\{G_i\}_{i=0}^{M-1}$ . If the set of values  $G_1, \dots, G_{M-1}$  has  $r_1$  values equal to  $G_0$ , then we should apply the positioning algorithm to calculate the position (rank) of  $G_0$  in this set.

Positioning algorithm

Step 1. Determine the capacity of subsets of ranks  $r_0 := |\{i | G_i < G_0\}|$ ,  $r_1 := |\{i | G_i = G_0\}|$

Step 2. If  $r_1 > 0$ , then select randomly in an equally probable way with probability  $1/(r_1 + 1)$  one of the numbers of the set  $\{1, \dots, r_1 + 1\}$  and denote it  $r_2$ . If  $r_1 = 0$ , then  $r_2 := 1$ .

Step 3. To designate as rank of values  $G_0$  in the set  $G_1, \dots, G_{M-1}$  the value  $\mathbf{R} = \mathbf{R}(G_1, \dots, G_{M-1}) := r_0 + r_2$ .

$H_0(\vartheta)$	$H_1(\vartheta)$	$H_2(\vartheta)$	$H_3(\vartheta)$	...	$H_{M-2}(\vartheta)$	$H_{M-1}(\vartheta)$
Rank $r = \mathcal{R}(\vartheta)$ is the position of $\ H_0(\vartheta)\ $ in a partially ranked set						
$\ H_{i_1}(\vartheta)\ $	$\ H_{i_2}(\vartheta)\ $	...	$\ H_0(\vartheta)\ $	...	$\ H_{i_{M-2}}(\vartheta)\ $	$\ H_{i_{M-1}}(\vartheta)\ $

### III. CONCLUSION

The paper deals with the problem of evaluating the effectiveness of the micro- (teacher, department), and meso-

(faculty, University) level of management by measuring the value of social capital. The possibility of using the modified sign-perturbed sums method for reducing the number of labor-intensive, long-term and costly measurements of social capital and ensuring high reliability prediction of the efficiency of University for a limited number of measurements is analyzed.

### ACKNOWLEDGMENTS

This work was financially supported by the Russian Foundation of Fundamental Researches (grant no. 18-013-00646/18).

### REFERENCES

- [1] Amelin K., Granichin O.N. *Randomized control strategies under arbitrary external noise*. IEEE Trans. Autom. Control 61, 2016, pp.1328–1333.
- [2] Capra, C. Monica and Lanier, Kelli and Meer, Shireen. *Attitudinal and Behavioral Measures of Trust: A New Comparison*, 2008, <https://www.gate.cnrs.fr/IMG/pdf/Capra.pdf>.
- [3] Carpenter, J. *Measuring social capital: Adding field experimental methods to the analytical toolbox*. Social capital and economic development, 2002.
- [4] Glaeser, E., Laibson D.I., Scheinkman J.A. and Sutter C.R. *Measuring trust*. The Quarterly Journal of Economics, Vol. 115 (3), 2000, pp. 811-846.
- [5] Granichin O.N. *Linear regression and filtering under nonstandard assumptions (Arbitrary noise)*. Trans. on AC, 2004, Vol.49. Oct., no.10, pp.1830-1835.
- [6] Paldam, M. *Social Capital: One or Many? Definition and Measurement*. Journal of Economic Surveys, Vol. 14 (Issue 5), 2000, p. 780.
- [7] Senov A., Amelin K., Amelina N., and Granichin O. *Exact confidence regions for linear regression parameter under external arbitrary noise*. in Proc. 2014 American Control Conference (ACC 2014), Portland, OR, June 4-6, 2014 (IEEE, Piscataway, NJ, 2014), pp. 5097–5102.
- [8] Senov A.A. and Granichin O.N. *Identification of linear regression parameters for arbitrary external noise in observations*. in Proc. 12th All-Russ. Meeting on Control Problems (VSPU-2014), Moscow, June 16–19, 2014 (Inst. Probl. Upr. im. V.A. Trapeznikova, Moscow, 2014), pp. 2708–2719.
- [9] Trapitsin, S.Y., Granichina O.A., Granichin O.N. *Information and mathematical models for evaluation of the effectiveness and quality of the University*. Proceedings of the 2017 International Conference "Quality Management, Transport and Information Security, Information Technologies", 2017, pp. 287-291.
- [10] Volkova M.V., Granichin O.N. Minimization of the functional of the medium risk type based on a finite (possibly small) set of experimental data // Stochastic optimization in computer science, 2017, Vol. 13, Issue. 2, pp. 3-37 (in Russian).