The wavelet solution for the linear first order integrodifferential equation of convolution type

I. A. Abdallah

Helwan University, Egypt

iaawavelets@yahoo.com

The numerical solution of a first order integro-differential equation of convolution type

$$f'(x) + P_0(x)\, f(x) - \int_0^x K_1(x-y)\, f(y)\, dy = g(x)$$

under the condition $f(0) = A$ is discussed. The method is based on the wavelet expansion analysis of functions.

Spline on the group $SU(2)$

E. Aghdassi†, S. F. Rzaev‡

University of Tabriz, Iran

†*esaghdassi@tabrizu.ac.ir*, ‡*rzseymur@hotmail.com*

Is known that, as a topological space, the group $SU(2)$ is homomorphic to the three-dimensional sphere $S^3 \subset R^4$. In particular $SU(2)$ is simply connected group Lie. In this point of view the announce of this paper is that the spline on the group $SU(2)$ may be formulated in a manner that closely parallels natural spline on the sphere. We make use of techniques developed by [1] and [2].

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Discrete counterparts of continuous-time additive Hopfield-type neural networks with impulses

Haydar Akça†, Rajai Alassar‡

King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia

†*akca@kfupm.edu.sa*, ‡*alassar@kfupm.edu.sa*

Discrete-time analogues of continuous-time additive Hopfield-type neural networks with impulses are formulated and their global stability characteristics are investigated.

In the modelling and analysis of dynamical phenomena various types of systems ranging downward in complexity from partial differential equations, functional differential equations, integro-differential equations, stochastic differential equations with hereditary term, difference equations and algebraic equations have been used. It is common to approximate models of higher levels of complexity by models of lower levels of complexity.

One of the most widely used techniques in the study of models involving ordinary differential equations is to approximate the system by means of a system of difference equations, whose solutions are expected to be samples of the solutions of differential equations at discrete instants of time as in the case of Euler-type methods and Runge–Kutta methods. It has been shown by several authors [4, 6] that the dynamics of numerical discretizations of differential equations can differ significantly from those of the original differential equations. Examples comparing the dynamics of the respective pairs of differential and difference equations are given in [5] (see also the introduction of our previous work [1]).

In [5] the global stability characteristic of a system of equations modelling the dynamics of additive Hopfield-type neural networks both in the continuous and discrete time cases is investigated. In particular, a novel method of obtaining a discrete-time dynamical system whose dynamics is inherited from the continuous-time dynamical system is studied. This aspect is important since numerical algorithms of Hopfield-type differential equations lead to discrete-time dynamic systems and such discrete-time systems should not give rise to any spurious behaviour if either system is to be used for coding equilibrium as associative memories corresponding to temporally uniform external stimuli obtained. The discrete-time models serve as global numerical methods on unbounded intervals for the continuous-time systems [3].

In our previous paper we investigated the global stability characteristics of these systems supplemented with impulse conditions in the continuous-time case. The presence of impulses required some modifications and the imposing of additional conditions on the systems. The present paper is devoted to the formulation of the discrete-time analogues of these impulsive systems and the investigation of their stability. Let us recall that convergent difference approximations for nonlinear impulsive systems of differential equations in a Banach space were obtained in [2].

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An adaptive wavelet scheme for numerical simulations of the domain walls motion in magnetics

R. N. Akhmadullin[†], B. N. Filippov[‡], E. M. Vaziev[†], V. V. Zverev[†,‡]

[†] *Ural State Technical University, Ekaterinburg, Russia,*
[‡] *Institute of Metal Physics, Ekaterinburg, Russia*
zverev@dpt.ustu.ru

A method for numerical simulations of the domain walls motion, based on *wavelet decompositions* is presented. Domain walls and other nonlinear formations (kinks, vortices, Bloch lines) in magnetics take the form of well localized patterns embedded into an almost uniform medium. This is just the case, when the adaptive multiresolution schemes, based on wavelets are more efficient [1]. This is due to the fact that if the pattern formation is spatially compact, a great many coefficients in wavelet decompositions are negligibly small, and the matrices become sparse.

We simulated numerically the translational motion and nonlinear oscillations of 180^0 domain wall in an infinite uniaxially anisotropic ferromagnetic [2, 3]. This rather idealized case is a good trial problem for examination of the new numerical schemes. It was assumed, that medium contains a single domain wall with an easy axis lying along the direction of a uniform dc magnetic field. The vectorial differential equations of motion were turned into the matrix equations by the use of the Daubechies wavelet decompositions [4, 5]. The derivation and the nonlinear operations were realized in wavelet bases as adaptive procedures for sparse matrices. We investigated the motion in a piecewise constant external magnetic field (a single step, periodic regimes). The matching of the different numerical schemes clearly indicates more high performance ot the wavelet approach.

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Polynomial wavelet-type expansions on the sphere

A. Askari Hemmat[†], M. Skopina[‡]

[†]*Valiasr University of Rafsanjan, Iran,*
[‡]*St. Petersburg State University, Russia*

[†]*askari321@yahoo.com*, [‡]*skopina@sk.usr.lgu.spb.su*

We present two polynomial systems. The first one is a Schauder basis for the space $C(S^d)$. Unfortunately, this basis is not suitable for implementations because of two drawbacks: the growth of the degree of polynomials is too rapid, no explicit formula for coefficient functional. The second one is a redundant wavelet-type system. The order of the growth of the degrees of polynomials is optimal. Any function in $C(S^d)$ can be expanded with respect to these wavelets so that a coefficient functional is the inner product of the function and the corresponding element of a a dual wavelet-type system. The dual wavelets are polynomials with the same growth of the degrees.

Earlier a similar wavelet construction was found by the second author for the case $d = 2$.

On extremal and approximation properties of shift-invariant spaces

V. F. Babenko[†,‡], A. A. Ligun[*,‡], A. A. Shumeiko[*]

[†]*Dnepropetrovsk National University, Ukraine,*
[‡]*Institute of Applied Mathematics and Mechanics NAS, Ukraine,*
[*]*Dneprodzerzhinsk State Technical University, Ukraine*

[†]*v_babenko@dp.ukrtel.net,* [‡]*isct@alicetele.com,*
[*]*shumeiko_a@ukr.net*

As usually we say that a space $\mathcal{S}^h$, $h > 0$, of complex-valued functions defined on $\mathbf{R}^d$ is shift-invariant if, for each $f \in \mathcal{S}^h$, the space $\mathcal{S}^h$ also contains the shifts $f(\cdot + h\alpha)$, $\alpha \in \mathbf{Z}^d$. We will be interested in the extremal and approximation properties of closed shift-invariant subspace of the space $H_p^\mu(\mathbf{R}^d)$ (see [1] for definition). We will be also interested in analogous subspaces of the spaces $H_p^\mu(\mathbf{T}^d)$ of periodic functions. Denote by $\mathcal{S}_0^h(\phi)$ the set of all finite linear combinations of functions of the form $\phi(\cdot + h\alpha)$. Its $H_p^\mu(\mathbf{R}^d)$ - closure we will denote by $\mathcal{S}_{p,\mu}^h(\phi)$. Such shift-invariant subspace generated by shifts of a single functions ϕ are called the principal shift-invariant subspace. Shift-invariant spaces are important in many areas of analysis (approximation theory, finite element analysis, wavelets) and were investigated by many authors (see [1]–[3]).

We intend to give a review of known results on extremal and approximation properties of closed shift-invariant subspaces of the spaces $H_p^\mu(\mathbf{R}^d)$ and $H_p^\mu(\mathbf{T}^d)$. We are also going to present some new results, especially for periodic functions. In particular we will present some exact inequalities of Bernstein–Nikol'skii type for functions from $\mathcal{S}^h_{p,\mu}(\phi)$ as wel as inequalities of Bohr–Favard–Wirtinger type for functions that are orthogonal to $\mathcal{S}^h_{p,\mu}(\phi)$. As a simple example of obtained inequality present here the following

Theorem. *Let $1 \le p < \infty$, and fuctions ϕ and λ, μ be such that the functions*

$$\Phi_p(\phi,\mu,\lambda;\omega) = \frac{\sum_{k\in\mathbf{Z}^d} |\mu(\omega+2\pi k)|^p |\hat{\phi}(\omega+2\pi k)|^p}{\sum_{k\in\mathbf{Z}^d} |\lambda(\omega+2\pi k)|^p |\hat{\phi}(\omega+2\pi k)|^p}$$

is continuous and bounded. Set $\Phi_p(\phi,\mu,\lambda) = \sup\limits_{\omega} \Phi_p(\phi,\mu,\lambda;\omega)^{1/p}$. Then for any $f \in \mathcal{S}^1_{p,\lambda}(\phi)$ the following best possible inequality holds

$$\|f\|_{p,\mu} \le \Phi_p(\phi,\mu,\lambda)\|f\|_{p,\lambda}.$$

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Non-separable wavelets and their application

V. F. Babenko[†,‡], A. A. Ligun[*,‡], A. A. Shumeiko[*]

[†]*Dnepropetrovsk National University, Ukraine,*
[‡]*Institute of Applied Mathematics and Mechanics NAS, Ukraine,*
[*]*Dneprodzerzhinsk State Technical University, Ukraine*

[†]*v_babenko@dp.ukrtel.net,* [‡]*isct@alicetele.com,*
[*]*shumeiko_a@ukr.net*

In article [1] is offered one method of the construction the local base function be based of the ideas subdivision. Constructed base function meet the demands for all characteristic of scaling functions. On its base is construct the sequence the discrete and continuous scaling functions to different polynomial accuracy, are construct scaling functions preserve average values on each stage.

The offered methods of the building the orthogonal four wavelets and pair biorthogonal four wavelets and, accordingly, wavelet filter. Develop the lifting-scheme of the constructing on base orthogonal and biorthogonal wavelets other orthogonal and biorthogonal four wavelets, preserve different additional characteristic.

Obtained wavelets are principal two-dimensional and can be not presented in the tensor product of the univariate wavelets.

Realize the procedure of the discrete wavelets-transformations. Construct as the cascade scheme as the scheme full wavelets-package, comprising of itself using different wavelets on each level of the decompositions.

On base non-separable wavelets-filters realize the computer program. Designed programme realization comprises of itself stage of preliminary preparation to filtering, filtering, preparation to quantization, several stages of the quantization (as scalar-quantization, so vector-quantization) and procedure of the lossless compression.

Thus realize full scheme of the coding and decoding images.

The results approve on test images. The preliminary investigations have shown that results commensurable with the best existing analogue.

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On the error evaluation of a multidimensional MRA based on polyharmonic splines

B. Bacchelli†, M. Bozzini‡, M. Rossini*

Dipartimento di Matematica e Applicazioni, Milano, Italy

†*bacchelli@matapp.unimib.it*, ‡*bozzini@matapp.unimib.it*, **rossini@matapp.unimib.it*

In this talk we discuss the error evaluation in a multiresolution process based on polyharmonic splines when a discrete signal is considered.

If the data are correct, the estimate is given by the convergence order of the considered MRA, otherwise we consider the noise contribution by studying the integrated mean square error. We provide also some numerical examples.

Semi-cardinal interpolation with box-splines on the three-direction mesh

Aurelian Bejancu

University of Leeds, United Kingdom

A.Bejancu@amsta.leeds.ac.uk

Interpolation at the vertices of a semi-plane lattice has recently been introduced as the bivariate equivalent of Schoenberg's univariate model of 'semi-cardinal interpolation', i. e. interpolation at the set of positive integers. In order to select a bivariate semi-cardinal interpolant from a box-spline space, suitable edge conditions need to be imposed. This talk will review the recent results on the construction and approximation order of semi-cardinal three-directional box-spline interpolation schemes based on extensions of the 'natural' and 'not-a-knot' end-conditions for univariate cubic splines.

Tight frame of co-rank one

M. S. Bespalov

Vladimir State University, Russia

bespalov-m2@vpti.vladimir.su

Жесткий фрейм ко-ранга один

В [1] как наглядный пример жесткого фрейма приводится система трех векторов в двумерном пространстве, иногда условно называемая «Мерседес»: $e_1 = (0,1)$, $e_2 = (-\frac{\sqrt{3}}{2}, -\frac{1}{2})$, $e_3 = (\frac{\sqrt{3}}{2}, -\frac{1}{2})$.

Тогда для любого $v \in H$ ($H = C^2$ или $H = R^2$):

$$v = \frac{2}{3}\sum_{k=1}^{3} c_k e_k, \text{ где } c_k = (v, e_k), \quad \text{и } ||v||_2^2 = \frac{2}{3}\sum_{k=1}^{3} |c_k|^2.$$

Данным свойством обладает любая система трех векторов в H, удовлетворяющая условиям $(e_k, e_j) = \frac{3}{2}\delta_{kj} - \frac{1}{2}$, $\sum_{k=1}^{3} e_k = 0$.

Чтобы первое условие было ближе к необходимому условию ортонормированного базиса ($(e_k, e_j) = \delta_{kj}$), «пронормируем» систему «Мерседес»: $d_k = \sqrt{\frac{2}{3}} e_k$. Тогда для любого $v \in H$:

$$v = \sum_{k=1}^{3} c_k d_k, \text{ где } c_k = (v, d_k), \quad \text{и } ||v||_2 = ||c||_2,$$

коэффициенты c_k обладают свойством минимальности: если $v = \sum_{k=1}^{3} c_k^* d_k$, и $c_k^* \neq c_k$, то $||v||_2 < ||c^*||_2$.

Если $H = C^n$ или $H = R^n$, то аналогичной системой $\{d_k\}_{k=1}^{n+1}$ служит система векторов $d_k = (a_1, a_2, \ldots, a_n)$, коэффициенты которых ($1 \leq k \leq n$):

$$a_i = 0 \text{ при } 0 \leq i \leq n-k, \quad a_{n-k+1} = \sqrt{\frac{n-k+1}{n-k+2}},$$

$$a_i = \frac{-1}{\sqrt{i(i+1)}} \text{ при } i \geq n-k+2.$$

Вектор d_{n+1} отличается от вектора d_n только знаком первой координаты.

Теорема. *Для данной системы* $(d_k, d_j) = \delta_{kj} - \frac{1}{n+1}$, $\sum_{k=1}^{n+1} d_k = 0$. *Для любого* $v \in H$: $v = \sum_{k=1}^{n+1} c_k d_k$, *где* $c_k = (v, d_k)$; $||v||_2 = ||c||_2$; *коэффициенты* c_k *обладают свойством минимальности: если* $v = \sum_{k=1}^{n+1} c_k^* d_k$, *и* $c_k^* \neq c_k$, *то* $||v||_2 < ||c^*||_2$.

Следовательно, $\{d_k\}_{k=1}^{n+1}$ жесткий фрейм с константой 1. Если $e_k = \sqrt{\frac{n+1}{n}} d_k$, то $|e_k| = 1$ и $\{e_k\}_{k=1}^{n+1}$ жесткий фрейм с константой $\frac{n+1}{n}$.

В трехмерном пространстве данный фрейм наглядно можно представить с помощью тетраэдра, центр которого совмещен с началом декартовой системы координат, вектор e_1 направлен по последней координате (вверх по высоте тетраэдра), проекция вектора e_2 на ортогональное дополнение к e_1 направлена по предпоследней координате. Этот же принцип положен в основу построения фрейма n-мерного пространства.

Утверждение. Не существует жесткого фрейма в трехмерном пространстве ко-ранга 2 (т. е. из пяти векторов) с условиями $(d_k, d_j) = \delta_{kj} - \alpha$, $\sum_{k=1}^{5} d_k = 0$.

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Criteria of weak generalized localization and Fourier–Walsh series of functions from Orlich classes

S. K. Bloshanskaya

Moscow Engineering Physics Institute (State University), Russia
i.bloshn@g23.relcom.ru

Критерии слабой обобщенной локализации и ряды Фурье–Уолша функций из классов Орлича[1]

Пусть $\mathfrak{A}$ — произвольное измеримое множество $\mathfrak{A} \subset I^N = [0,1)^N$, $N \geq 1$, $\mu\mathfrak{A} > 0$ (μ — мера Лебега). Ранее (см. [1, 2]) нами исследовался вопрос о сходимости почти всюду (п.в.) кратных рядов Фурье–Уолша (суммируемых по прямоугольникам) функций $f \in L_p$, $p \geq 1$, равных нулю на $\mathfrak{A}$. Точнее, нас интересовал вопрос о сходимости п.в. указанных рядов либо на

[1]Работа выполнена при финансовой поддержке РФФИ (проект 02-01-00428).

множестве $\mathfrak{A}$ (обобщенная локализация п.в.), либо на каких-либо его подмножествах $\mathfrak{A}_1 \subset \mathfrak{A}$, $\mu\mathfrak{A}_1 > 0$ — слабая обобщенная локализация п.в. (СОЛ).

Рассмотрим понятие свойства B_k, $k = 1, 2$, множества $\mathfrak{A}$. Пусть $J_k = \{j_1, j_k\} \subset M = \{1, 2, \ldots, N\}$, $1 \leq j_1 < j_k \leq N$, и пусть Ω_{J_k} — произвольное (непустое) открытое множество $\Omega_{J_k} \subset I_{J_k} = \{(x_{j_1}, x_{j_k}) : \ 0 \leq x_{j_s} < 1, \ s = 1, k\} \subset I^N$. Положим $W_{J_k} = \Omega_{J_k} \times I_{J_{N-k}}$, $W_k = \bigcup_{J_k \subset M} W_{J_k}$ и (предполагая при $k = 2$, что $W_k^0 \neq \emptyset$) $W_k^0 = \bigcap_{J_k \subset M} W_{J_k}$.

Для широкого класса множеств $\mathfrak{A}$ нами было доказано (см. [2]), что:

1) СОЛ в классе $L_1(I^N)$, $N \geq 1$, справедлива на множестве $\mathfrak{A}$ тогда и только тогда, когда $\mathfrak{A}$ обладает свойством B_1;

2) СОЛ в классах $L_p(I^N)$, $p > 1$, $N \geq 2$, справедлива на $\mathfrak{A}$ тогда и только тогда, когда $\mathfrak{A}$ обладает свойством B_2.

В частности, показано, что для любой функции $f \in L_p$, $f(x) = 0$ на $\mathfrak{A}$ кратный ряд Фурье–Уолша (суммируемый по прямоугольникам) сходится п.в. к нулю на множестве W_k^0, $k = k(p, N)$, тогда и только тогда, когда $\mathfrak{A}$ обладает свойством B_k (причем $k = 1$, если $p = 1$, $N \geq 1$, и $k = 2$, если $p > 1$, $N \geq 2$).

Таким образом, справедливость СОЛ (для кратных рядов Фурье–Уолша суммируемых по прямоугольникам) на множестве $\mathfrak{A}$ в классах $L_p, p \geq 1$ существенно зависит от степени суммируемости разлагаемой функции, а также от структуры и геометрии $\mathfrak{A}$.

В настоящей работе мы исследуем вопрос о справедливости СОЛ для кратных рядов Фурье–Уолша в классах, «лежащих между» L_1 и L_p, $p > 1$. Точнее, мы даем ответ на вопрос: в каких классах $\Phi_2(L)$ ($\Phi_2(u) = o(u^p)$ при $u \to \infty$, $p > 1$), гарантирующих справедливость СОЛ на измеримом множестве $\mathfrak{A}$, структура и геометрия этого множества должны определяться еще свойством B_2, а в каких классах $\Phi_1(L)$ ($\Phi_1(u) = o(\Phi_2(u))$ при $u \to \infty$) для справедливости СОЛ на $\mathfrak{A}$ структура и геометрия этого множества должны определяться уже «более обременительным» свойством B_1?

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Structural and geometric characteristics of convergence and divergence sets for multiple Fourier series of functions which are equal to zero on some set

I. L. Bloshanskii

Moscow State Open University, Russia

i.bloshn@g23.relcom.ru

Структурные и геометрические характеристики множеств сходимости и расходимости кратных рядов Фурье функций, равных нулю на некотором множестве[2]

1. Пусть E — произвольное измеримое множество $E \subset T^N = [-\pi, \pi)^N$, $N \geq 1$, $\mu E > 0$ (μ — мера Лебега), и пусть $\mathcal{A} = \mathcal{A}(T^N)$ некоторое линейное подпространство $L_1(T^N)$. Нас интересует вопрос о сходимости на множестве E (почти всюду (п.в.) или в каждой точке) кратного ряда Фурье функции f, $f \in \mathcal{A}$, $f(x) = 0$ на E в зависимости от гладкости функции f

[2]Работа выполнена при финансовой поддержке РФФИ (проект 02-01-00428).

(т.е. от вида (определения) пространства $\mathcal{A}$) и от *структурно-геометрических характеристик* множества E ($СГХ(E)$). Таким образом, наша цель дать описание пары ($\mathcal{A}$, $СГХ(E)$). Поставленную задачу оказалось удобно сформулировать и исследовать в терминах *обобщенной локализации почти всюду* (ОЛ) и *слабой обобщенной локализации почти всюду* (СОЛ). Последнее означает, что для кратного ряда Фурье функции f, равной нулю на множестве E, исследуется вопрос о сходимости п.в. либо на множестве E (ОЛ), либо на каких-либо его подмножествах $E_1 \subset E$, $\mu E_1 > 0$ (СОЛ).

2. Обозначим через $\mathbb{F} = \left\{\mathcal{A}_k^{(j)}\right\}_{k,j}$ матрицу $N \times M$ ($N \geq 2$, $M \geq 6$), элементами которой являются функциональные пространства $\mathcal{A} = \mathcal{A}_k^{(j)}$, $k = 1, \ldots, N$, $j = 1, \ldots, M$. При этом пространство $\mathcal{A}_k^{(j)}$ либо L_1, либо некоторое линейное подпространство L_1. Например, $\mathcal{A}_1^{(1)} = \mathcal{A}_1^{(2)} = L_1$; $\mathcal{A}_2^{(2)} = L_\infty$; $\mathcal{A}_2^{(4)} = L_2$; $\mathcal{A}_2^{(5)} = L_p$, $1 < p < 2$; $\mathcal{A}_2^{(6)} = L(\log^+ L)^2$; $\mathcal{A}_k^{(2)} = H^{\omega^{(k)}}$, где $\omega^{(k)} = \omega^{(k)}(\delta)$ — некоторые модули непрерывности.

3. Далее, для каждого k, $1 \leq k \leq N$, определим некоторые структурно- геометрические характеристики множества E, которые мы будем называть $\mathbb{B}_k$ свойством множества E. Указанные характеристики множества E определяются существованием у E открытых п.в. (открытых с точностью до множества меры нуль) подмножеств Ω, имеющих определенную «крестообразную» геометрию, связанную с ортогональными проекциями некоторого подмножества $\Omega_1 \subset \Omega$ на k - мерные (если $k < N$) координатные плоскости. При $k = N$ множество E, обладающее свойством $\mathbb{B}_N$, это множество, для которого существует открытое п.в. подмножество.

4. Нами сформулированы результаты, описывающие взаимосвязь между «гладкостью» функции f (в рамках функциональных пространств матрицы $\mathbb{F}$) и структурно-геометрическими характеристиками множества E ($СГХ(E)$), на котором f равна нулю, для кратных тригонометрических рядов Фурье, суммируемых по прямоугольникам. Данная взаимосвязь описывается

парой $(\mathcal{A}_r^{(j)}, \mathbb{B}_k)$, $1 \le r, k \le N$, $j = 1, \ldots, M$.

Nonlinear approximation: an overview

Yu. Brudnyi

Israel Institute of Technology, Haifa, Israel
ybrudnyi@math.technion.ac.il

The talk presents some results on quantitative nonlinear approximation by libraries generated by B-splines, elemetary rational functions and general refinable functions of finitely supported mask. Several geometric and graph theoretic problems related to adaptive approximation algorithms involved in proofs will be discussed as well.

Numerical experiments on bivariate polynomial interpolation at new nodal sets[3]

M. Caliari[†], S. De Marchi[‡], M. Vianello[*]

[†,‡] *University of Verona, Italy,*
[*] *University of Padova, Italy*

[†]*caliari@sci.univr.it,* [‡]*stefano.demarchi@univr.it,*
[*]*marcov@math.unipd.it*

As known, the problem of choosing "good" nodes is a central one in polynomial interpolation. While the problem appears essentially solved in one dimension as for uniform approximation (all good nodal sequences are asymptotically equidistributed with respect to the arccosine metric), in several variables it still represents a substantially open question.

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In this work we experiment new nodal sets for bivariate polynomial interpolation on the square and on the disk. Numerical evaluations of the Lebesgue constants are provided as well as the approximation behavior on some test functions. In particular, we consider points which are asymptotically equidistributed with respect to the *Dubiner metric* [4], which encapsulates the local properties of polynomial spaces on a given multivariate compact set, and in one dimension coincides with the arccosine metric. Bivariate polynomial interpolation is here constructed by means of the *Newton-like algorithm* recently proposed by Sauer [5]. Another nodal set is given by *tensor-product fast Leja points* for tensor-product interpolation on the square (cf. [1]). Indeed, it has been recently proved that tensor-product Gauss–Lobatto points, which are asymptotically distributed as the Leja points in one dimension, are Fekete points for the cube [2]. As known, Leja points are computationally effective, since they give an increasing family of nodal sets with respect to the degree, and they stabilize Newton interpolation. Finally, we present an application of bivariate polynomial interpolation to the compression and *fast evaluation of bivariate integral transforms*, extending our previous work on the subject [3].

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Description of seismic events using wavelet transform

Nina Častová†, Hana Doležalová‡, David Horák*, Zdeněk Kaláb**

†,* *VSB Technical University of Ostrava, Czech Republic,*
‡,** *Institute of Geonics, Academy of Sciences of the Czech Republic*

† *nina.castova@vsb.cz,* ‡ *dolezalo@ugn.cas.cz,* * *david.horak@vsb.cz,*
** *kalab@ugn.cas.cz*

In this paper, real seismological signals are used for processing. These data represent mining induced seismic events recorded at surface seismic station in the Karviná region (Upper Silesian Coal Basin, Czech Republic) during mine activity in one coalface with very complicated geological and geomechanical conditions. Results from vertical component are presented as example here although all three components were interpreted. These signals from the mathematical point of view are possible to be defined as slight signals, e.i. their formal writings are $X = X_r + \varepsilon$, where X_r represents seismological signal and ε is Gaussian white noise with normal distribution $N(0, \sigma^2)$. This seismological signal is of the same order as noise $X_r(t) \sim \varepsilon(t)$. This signal fulfils the basic conditions for presented mathematical processing.

Methodology for processing of these slight signals using wavelet transform is presented in this paper. Briefly, three basic steps describe this procedure:

1. Selection of optimal wavelet and optimal wavelet basis B^{opt} (Daubechies real and complex orthogonal wavelets, symmetric complex wavelets) for selected data set based on minimal entropy: $B^{opt} = \arg\min_B E(X, B)$. The best results were

reached by symmetric complex wavelets with scaling coefficients SCD-6:

$$h(SCD6) = \frac{1}{64}\,(-3 - i\sqrt{15}, 5 - i\sqrt{15}, 30 + i2\sqrt{15},\\ 30 + i2\sqrt{15}, 5 - i\sqrt{15}, -3 - i\sqrt{15}).$$

2. Wavelet packed decomposition and filtration of data using universal criterion of thresholding of the form $\lambda = \sigma\sqrt{2\ln(n)}$, where σ is minimal variance of the sum of packets decomposition of chosen level.

3. Cluster analysis of decomposed data using eigenvalues of variance matrices, singular value of packet decompositions and eigenvalues of cosine matrix containing values of cosines of angles among vectors: $cos\,\varphi = \frac{(x_i,x_j)}{\|x_i\|\|x_j\|}$, where x_i, x_j denote denoised signals ($i, j = 1, 2, \ldots, N$; N is the number of input signals) or packet decompositions of chosen level, respectively. Selected data set contains 36 seismic events that are possible to be divided into six groups according to their wave patterns, i.e. source mechanisms (local geology is not included one source area, one seismic station). All programs were elaborated using program MATLAB 5.

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Haar wavelets matrix factorization in wave propagation

Carlo Cattani

University of Rome "La Sapienza", Italy

carlo.cattani@uniroma1.it

In order to extend the physical attitude of wavelets, the classical wave equation in dispersive medium (Klein–Gordon equation) will be considered. The dispersive wave solution of the propagation equation will be interpreted as a superposition of "small" waves compactly supported in the space of the physical variable (time) and in the space of the frequency domain (angular frequency).

In particular this suggests us to investigate the wave propagation at a given resolution. That is, it will be considered the discrete wavelet approach, and the solution will be expressed by a truncated wavelet series. The series coefficients are determined as solution of ordinary differential equations, whose construction follows from the explicit computation of the Haar wavelet connection coefficients.

In particular, let us consider the Klein–Gordon equation, in absence of sources, in the unknown function

$$u(x,t)\colon (\mathbb{R} \subset L^2(\mathbb{R})) \times [0,T] \to \mathbb{R},$$

$$\left(\rho\frac{\partial^2}{\partial t^2} - a^2\frac{\partial^2}{\partial x^2} \pm b^2\right) u = 0, \qquad x \in \mathbb{R},\ t \in (0,T],\ T \leq \infty,$$

where the constant values $\rho,\ a,\ b \in \mathbb{R}$ are given.

In order to make some physical applications, with fixed initial-boundary conditions, the approximate Haar wavelet solutions of the Klein–Gordon equation will be computed as

$$u(x,t) = \sum_{n=0}^{2^N-1} \sum_{k=0}^{2^n-1} \beta_k^n(x)\psi_k^n(t),$$

$\psi_k^n(t)$ being the Haar wavelets.

Since the Haar functions $\psi_k^n(t)$ are not differentiable functions, in order to explicitly compute the connection coefficients, the derivative of the Haar wavelets is performed by a numerical algorithm, through a smooth interpolation of points. It is shown that, with the simplest interpolation, it is possible to define a linear operator which allows the derivatives of the Haar wavelets. In case of highly localized (piecewise constant) initial functions or initial functions with sharp jumps, the Haar basis might strongly reduce the number of wavelet coefficients. Finally, the complexity of the Haar wavelet transform is also reduced by a slicing algorithm. With this algorithm, the Haar transform follows from a simple composition of Jordan-block matrices. Thus the wavelet solution of the Klein–Gordon, at a fixed scale resolution, is easily obtained by transforming the differential operator into a simple product of sparse matrices.

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Wavelets and splines for vertical scratches removal in old films

Patrizia Ciarlini[†], Giuseppe Costanzo[‡],
Maria Laura Lo Cascio[*]

[†]*Istituto per le Applicazioni del Calcolo, Roma, Italy,*
[*]*Università di Messina, Italy*

[†]*ciarlini@iac.rm.cnr.it*, [‡]*gcostanzo99@hotmail.com*,
[*]*locascio@dipmat.unime.it*

Hystorical movies are often damaged by many different factor and the interest in restoring them has been growing. Vertical scratches are a very frequent defect, characterized by the fact that they has a typical line-like profile, which extends from the top to the bottom of the image, and they are visible, as bright or dark lines, at the same location of several subsequent frames. Since the manual restoration is to much expensive both in time and money, semi-automatic techniques working on digitalized images are highly desiderable.

We introduce a method to restore a vertical scratch in a given position. It is based on a wavelet decomposition by biorthogonal bases, instead of the Haar bases used in [1], and on spline approximations of the filtered image. Particular attention has been devoted to identify the spline parameters in connection to the properties of the line-scratch and of the image texture in a suitable neigbourhood of the scratch itself. An analysis of the scratch region texture is needed to reconstruct the homogeneous zones without removing high frequency details. The procedure has been validated as applied to monocromatic images with simulated scratches. The good performances of the method are shown in several old damaged images and simulated examples.

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On a new class of bivariate convolved refinable functions

Costanza Conti[†], Laura Gori[‡], Francesca Pitolli[*]

[†] *Università di Firenze, Italy,*
[‡,*] *Università di Roma "La Sapienza", Italy*

[†] *costanza@sirio.de.unifi.it,* [‡] *gori@dmmm.uniroma1.it,*
[*] *pitolli@dmmm.uniroma1.it*

In this talk a new class of bivariate refinable functions is presented and some of its properties are investigated. The new class is constructed by convolving the tensor product of two univariate refinable functions belonging to a particular family with a third one in the same family. As in the case of the box splines, the convolution product is the directional convolution product.

On a new class of polynomial spline functions

Paolo Costantini

Dipartimento di Scienze Matematiche ed Informatiche "R. Magari", Siena, Italy

costantini@unisi.it

An analysis of the methods recently proposed for the construction of curves and surfaces reveals a particular attention to the control of the shape, both in free-form design (where it is necessary to easily modify the designed objects) and in interpolation or approximation (where the curve or the surface must often inherit the geometric characteristics of the data set). Moreover, it appears that a conspicuous part of the corresponding mathematical tools are based on new, more flexible spaces of functions, which permit an adaptive selection of their shape.

A typical and well-known example is given by *tension splines*, where the cubic polynomial splines generated by the space

$$\mathbf{P}_3 = \mathrm{span}\{\mathbf{P}_1, x^2, x^3\}$$

are substituted by piecewise functions generated by spaces of the form $\mathrm{span}\{\mathbf{P}_1, \Phi, \Psi\}$. For limit values of the *tension parameters* the contribution of Φ and Ψ vanishes, the spaces tend to $\mathbf{P}_1$ and the functions tend to assume a piecewise linear shape.

Although tension splines constitute undoubtedly a basic tool in one dimensional and tensor-product interpolation, the limit shape is, in some case, not completely satisfactory. For instance, the limit of a tensor-product surface generated by tension splines has a piecewise bilinear behavior, and this is not always appropriate for approximating a data set sampled from a given object. Therefore, it would be useful to enlarge the limit space and consequently augment the flexibility of the shape of the limit functions.

Let $t_0 < t_1 < \ldots < t_n$ be a knot sequence, let $h_0, h_1, \ldots, h_{n-1}$ be the corresponding spacing and let $k_0, k_1, \ldots, k_n$, $k_i \geq 4$, be a sequence of integers. Aim of this talk is to discuss the properties of C^r, $r = 2, 3$, variable degree polynomial splines generated by the spaces

$$\mathrm{span}\{\mathbf{P}_2, ((t_{i+1} - t)/h_i)^{k_i}, ((t - t_i)/h_i)^{k_{i+1}}\},$$

which, for limit values of the k_i, tend to C^{r-2} quadratic polynomial splines. Their applications in spatial data approximation will also be discussed.

Error analysis for quadratic spline quasi-interpolants on non-uniform criss-cross triangulations of bounded rectangular domains

Catterina Dagnino[†], Paul Sablonnière[‡]

[†] *University of Torino, Italy,*
[‡,*] *INSA, Centre de Mathématiques, Rennes Cédex, France*
[†] *catterina.dagnino@unito.it,* [‡] *Paul.Sablonniere@insa-rennes.fr*

We consider some C^1 quadratic spline quasi-interpolants on a non-uniform criss-cross triangulation of a bounded rectangular domain Ω.

These operators are of the form

$$Qf(x,y) = \sum_{(i,j)\in K(\Omega)} \mu_{i,j}(f) B_{i,j}(x,y),$$

where $K(\Omega)$ is the set of indices of B-splines B_{ij} whose support is included in the domain Ω (see [3]) and $\mu_{ij}(f)$ is a discrete linear functional based on values of f in a neighbourhood of $(x_i, y_j) \epsilon$ supp (B_{ij}) (see [2]). The data points (x_r, y_s) are vertices of the triangulation of the domain Ω, where the function f is to be approximated.

By means of a procedure similar to the one followed in [1] for the quasi-interpolants defined in [4], we show that the operators here studied can approximate a real function and its partial derivatives up to an optimal order and we derive local and global upper bounds for the errors.

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Approximation of scattered data by direct extension of local approximations

Oleg Davydov

Mathematisches Institut, Universität Giessen, Germany
oleg.davydov@math.uni-giessen.de

We discuss a recently devloped "direct extension" method for scattered data fitting, where reliable discrete least squares approximations (polynomial or non-polynomial) to the local fragments of the data are computed in the first stage with the control of the norming constant, and then they are extended to a smooth Bézier surface, with linear total computational cost.

Local hybrid approximation for scattered data fitting based on splines

O. Davydov†, R. Morandi‡, A. Sestini*

†*Justus-Liebig-Universität Giessen, Germany,*
‡,**Università di Firenze, Italy*

†*oleg.davydov@math.uni-giessen.de*, ‡*morandi@de.unifi.it*,
**sestini@de.unifi.it*

As well as in [1, 2], we consider scattered data approximation based on a two–stage method which first generates and then uses local approximations for defining the final approximation as a C^1 or C^2 spline surface on a uniform four–directional mesh covering the domain. In this work a local hybrid approximation scheme based on polynomials and radial basis functions is tested for the first stage. An ascending adaptive procedure for knot positioning is given, [3].

Based on experiments related to several real–world data sets, the numerical results confirm that high quality and artifact–free approximating surfaces can be obtained with this approach, while keeping linear its computational complexity.

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Polynomial wavelet-type expansions on the ball

M. A. Dehghan[†], M. Skopina[‡]

[†] *Valiasr University of Rafsanjan, Iran,*
[‡] *St. Petersburg State University, Russia*

[†] *d_m_dehghan@yahoo.com*, [‡] *skopina@sk.usr.lgu.spb.su*

Our goal is finding polynomial expansions of continuous functions on the d-dimentional ball B^d. Wavelet technique is useful for problems of such kind. In particular, the second author found optimal polynomial orthogonal bases for $C[a, b]$ with help of a multiresolution construction with respect to some special "shift"-operators. Though a similar construction can be realized in any Hilbert space with an initial orthogonal polynomial basis, it is impossible to provide a Schauder basis for $C(B^d)$ in this way. Instead, we suggest a redundant wavelet-type system so that any function in $C(B^d)$ can be expanded with respect to these wavelets, and a coefficient functional is the inner product of the function and the corresponding element of a dual wavelet-type system. Both the systems consist of ridge polynomials such that the order of the growth of their degrees is optimal.

Earlier a similar construction was found by the second author for the case $d = 2$. A new technique has been used to extend that result to the multidimentional case.

Wavelet expansion in the case of irregular nets[4]

Yu. K. Demyanovich

St. Petersburg State University, Russia
Yuri.Demjanovich@paloma.spbu.ru

There are many works devoted to wavelet expansions on the equidistant nets (see [1]–[4] and bibliography mentioned there). In practice an irregular net may be useful in the case of nonregular data processing for improvement of approximation quality without complication of calculations.

The aim of this talk is introduction of wavelet expansions on the irregular net with finite or infinite points of condensation.

Suppose $\{\widetilde{X}_i\}_{i\in\mathcal{Z}}$ is a family of spline spaces $\widetilde{X}_i$, inclusions $\widetilde{X}_i \subset \widetilde{X}_{i+1}$ are valid, and linear functionals $f_{ij} \in \widetilde{X}^*_{i+1}$ are biorthogonal to the basic functions $\varphi_{ij'}$ of the space $\widetilde{X}_i$, so that $f_{ij}(\varphi_{ij'}) = \delta_{jj'}, \quad i, j, j' \in \mathcal{Z}$; here $\mathcal{Z}$ is the set of integers.

Let P_i is project operator defined with formula

$$P_i\tilde{u} \overset{\mathrm{def}}{=} \sum_j f_{ij}(\tilde{u})\varphi_{ij},\ \forall\tilde{u} \in \widetilde{X}_{i+1}, \qquad \widetilde{W}_i \overset{\mathrm{def}}{=} (I - P_i)\widetilde{X}_{i+1}. \quad (1)$$

It gives direct sum (wavelet expansion) $\widetilde{X}_{i+1} = \widetilde{X}_i \dot{+} \widetilde{W}_i$, where $\widetilde{W}_i$ is wavelet space, $i \in \mathcal{Z}$.

In the case of equidistant net the embedding of the minimal spline spaces (see [5]) and biorthogonal systems are investigated in [6]–[7] with introducing of pseudo-convolution algebra; but such approach is insufficient in that case.

In this talk we consider certain problems related to the construction of the chains of embedded minimal spline spaces and the tracing of the simple realization of biorthogonal functionals for basic splines. As a result everybody can see the wavelet expansion in the form (1) and direct solutions of new interpolation problems in

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the spaces of mentioned splines (including B-splines) on arbitrary irregular net (see also [8]).

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Finite-element approximation on manifolds[5]

Yu. K. Demyanovich

St. Petersburg State University, Russia

Yuri.Demjanovich@paloma.spbu.ru

The finite Courant's element, Zlamal's element, Argyris' element, Bell's one and a lot of other plane local approximations are well-known (see [1–4]). The author discussed local approximations on smooth manifolds without boundary (see [5] and bibliography given there). The aim of presented work is construction of similar approximations for enough wide class of smooth manifolds including manifolds with boundary.

Let $\Omega_j(x)$ are summable functions with compact support in n-dimensional space $\mathcal{R}^n$ where index j runs through some set J. We suppose the quantity of elements in the set $\{j|x \in \mathrm{supp}\Omega_j\}$ is finite and uniformly bounded as to $x \in \mathcal{R}^n$. Let's assume the conditions

$$\sum_j x_j^\alpha \Omega_j(x) = x^\alpha, \quad |\alpha| \le m,$$

are right; here $\{x_j\}$ is some net of knots in $\mathcal{R}^n$, $\alpha = (\alpha_1, \dots, \alpha_n)$, α_i and m are nonnegative integer numbers, $x = (x^{(1)}, \dots, x^{(n)})$, $x^{(i)} \in \mathcal{R}^1$, $i = 1, \dots, n$, $x^\alpha = (x^{(1)})^{\alpha_1} \dots x^{(n)})^{\alpha_n}$, $|\alpha| = \sum_{i=1}^n \alpha_i$. Linear combinations of functions Ω_j,

$$\widetilde{U}(x) = \sum_j c_j \Omega_j(x), \quad x \in \mathcal{R}^n, \quad c_j \in \mathcal{R}^1, \tag{1}$$

are called the local (or finite-element) approximations (see [1–4]). The same ones could be discussed in half-space $\overline{\mathcal{R}}_+^n \stackrel{\text{def}}{=} \{x|x \in \mathcal{R}^n, x^{(n)} \ge 0\}$.

The approximations of Courant, Zlamal, Argyris etc. might be written in the form (1). The same constructions for functions defined on manifolds (for instance, on the surfaces) collide with great

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obstacles. In particular a surface is decomposed in finite number of simple surfaces and its own approximation is discussed on the each of them; however it is very difficult to stick the basic functions for adjoining simple surfaces with preservation of properties of approach, smoothness and stability.

These obstacles are overcome by usage of atlas with infinite (continual) quantity of maps and triangulation of the manifold; last ones generate a special transformation of $\Omega_j(x)$ for construction of basic functions on the manifold. Here we discuss approximation in the form (1) for functions defined on smooth manifold, and we investigate the quality of approach and stability for respective algorithm. Realization of the algorithm is very simple, it may be easy parallelized, and it doesn't need in calculations of curvilinear elements of the triangulation.

The proposed algorithm (for $n = 2$ and $m = 1$) had been realized with computers for approximation of functions defined on the sphere, on the torus and on the sphere with two handles.

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Frames and bases generated by Fourier window transform[6]

V. L. Dol'nikov, N. A. Strelkov

Yaroslavl State University, Russia
dolnikov@uniyar.ac.ru

Let $\mathbb{A}$ and $\mathbb{B}$ be two lattices in $\mathbb{R}^n$ and let

$$\{\varphi_{a,b}\}_{a\in\mathbb{A},b\in\mathbb{B}} \tag{1}$$

be the family of functions

$$\varphi_{a,b}(x) = e^{i(a,x)}\varphi(x-b),$$

where $\varphi = \chi_\Omega$ is the characteristic function of a measurable set $\Omega \subset \mathbb{R}^n$.

In terms of geometrical properties of the triplet $\{\Omega, \mathbb{A}, \mathbb{B}\}$ (packing, covering, tiling, and so on) we shall give a complete description of systems (1) which are

(i) complete,

(ii) frames,

(iii) tight frames,

(iv) bases

in $L_2(\mathbb{R}^n)$.

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Non-linear 4-point subdivision schemes

Nira Dyn

Tel Aviv University, Israel

niradyn@math.tau.ac.il

In this talk two families of non-linear univariate interpolatory subdivision schemes are discussed. All schemes in these families are extensions of the "classical" linear interpolatory 4-point scheme. The schemes in the first family are designed for the generation of curves from control points, and the schemes in the second family operate on functional data and generate functions. Linear subdivision schemes use a fixed refinement rule to refine repeatedly the data. Due to the limited quality of the curves generated by linear schemes, various attempts have been made at designing non-linear and geometry dependent schemes, which adapt to the geometry of the data. Here we present one attempt at modifying the 4-point scheme, by allowing a variable tension parameter, in cotrast to the fixed tension parameter in the linear scheme. The tension parameter is adapted locally according to the geometry of the control polygon within the 4-point stencil. With this change the new schemes retain the locality of the 4-point scheme, and at the same time achieve important shape-preservation properties, such as artifact elimination and co-convexity preservation. The second family of schemes is designed for the interpolation of equidistant data, sampled from piecewise smooth functions. These schemes are based on the ENO interpolation technique, and use a 4-point stencil for the insertion of a new point. The stencil is either centered relative to the inserted point or moved by one point to one of the two sides, or is a weighted sum of the three stencils. In contrast to the linear case where the stencil is always centered, the choice of the stencil in the new schemes is data dependent and aims at using data from the smooth parts of the sampled function. These schemes converge to Holder continuous limit functions, and reproduce cubic polynomials. Such schemes were found effective for image compression.

This is a report on joint works with A. Cohen, D. Levin, M. Marinov, and B. Matei.

On the construction of orthogonal wavelets

Yu. A. Farkov

Moscow State Geological Prospecting University, Russia

farkov@msgpa.msgpa.ru

This talk is devoted to an orthogonal wavelet constructions on the positive semiaxis $\mathbf{R}_+ = [0, +\infty)$. Let p be a fixed natural number greater than 1. For $x \in \mathbf{R}_+$ and any positive integer j we set

$$x_j = [p^j x] (\operatorname{mod} p), \qquad x_{-j} = [p^{1-j} x] (\operatorname{mod} p), \tag{1}$$

where $[x]$ is the integer part of x. Note that $x_{-j} = 0$ for $j \geq k = k(x)$. Using (1) for $x, \omega \in \mathbf{R}_+$, we define

$$\chi(x, \omega) = \exp\Big(\frac{2\pi i}{p} \sum_{j=1}^{\infty} (x_j \omega_{-j} + x_{-j} \omega_j)\Big).$$

For $f \in L^1(\mathbf{R}_+)$ the Walsh–Fourier transform is defined by

$$\tilde{f}(\omega) = \int_{\mathbf{R}_+} f(x) \overline{\chi(x, \omega)} dx.$$

And if $f \in L^2(\mathbf{R}_+)$, then $\tilde{f}$ is defined as the limit in $L^2(\mathbf{R}_+)$ of the suitable function sequence (see, e.g., [1]). For $x, y, z \in \mathbf{R}_+$ we write $x \oplus y = z$, if $x_j + y_j = z_j \, (\operatorname{mod} p)$ $(j \in \mathbf{Z} \setminus \{0\})$ where x_j, y_j, z_j are defined by (1).

In particular, we prove the following:

Theorem. *Let*

$$m_0(\omega) = \sum_{\alpha=0}^{p^n - 1} a_\alpha \overline{\chi(\alpha, \omega)}$$

be a polynomial satisfying the following conditions:

(a) $m_0(0) = 1$ *and* $m_0(\omega) \neq 0$ *for all* $\omega \in [0, 1/p)$.

(b) $\sum_{j=0}^{p-1} |m_0(sp^{-n} \oplus j)|^2 = 1$ *for* $s = 0, 1, \dots, p^n - 1$.

If the Walsh–Fourier transform of $\varphi \in L^2(\mathbf{R}_+)$ *can be written as*

$$\tilde{\varphi}(\omega) = \prod_{j=1}^{\infty} m_0(p^{-j}\omega),$$

then φ *is an orthogonal scaling function in* $L^2(\mathbf{R}_+)$.

This result with the general scheme described in [2] provides sufficient information for constructing orthogonal wavelet bases in $L^2(\mathbf{R}_+)$. Several examples will be given.

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MRA on a locally compact abelian group

M. Fashandi

Ferdowsi University, Mashad, Iran

Fashandi@math.um.ac.ir

Given a locally compact semidirect group $G = S\sigma T$, with S abelian, we study the canonical representation of G on $L^2(S)$, which is the main tool for constructing a Multiresolution Analysis on $L^2(S)$.

Interpolation of Banach spaces by means of Gabor-wavelets frames[7]

Massimo Fornasier

University of Vienna, Austria

mfornasi@math.unipd.it

The quality of the analysis and the approximation properties of a frame can be characterized by the norm of those Banach function spaces the frame is able to reproduce [1]. Conversely, associated to a frame one can always construct a corresponding family of Banach spaces [2, 5, 7]. Thus, one can investigate how to *interpolate* approximation properties and Banach spaces by mixing corresponding bases in a suitable way. This kind of interpolation should be distinguished from the well known complex and real methods, since the former can be in general *non-linear*. In particular, we shall show that a suitable non-linear parametric combination of Gabor and wavelet frames [4] can define a new family of Banach spaces. By construction, they can be considered as intermediate between modulation [2] and Besov [5] spaces. It turns out that these *interpolation* spaces are the so called α-modulation spaces, already introduced by Gröbner and Feichtinger in 1992 [3, 6]. Our approach is based on the L^2 theory developed in [4] and on a generalization of the localization of frame theory, recently introduced by Gröchenig [7].

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Dyadic analogue of tauberian theorem of Wiener and related topics

B. I. Golubov

Moscow Engineering Physics Institute (State University), Russia

golubov@mail.mipt.ru

For $x \in R_+$ and natural n we set $x_n \equiv [2^n x] \pmod 2$, $x_{-n} \equiv [2^{1-n}x] \pmod 2$. For $x,\ y \in R_+$ let us set $x \oplus y = z$, where the number $z \in R_+$ has dyadic digits $z_n = x_n + y_n \pmod 2$, $n \in Z \setminus \{0\}$. Let $\tilde{f}$ be Walsh transform of the function $f \in L\,(R_+)$ or $f \in L^2\,(R_+)$ (see [1] , chapter 1).

For two functions $f,\, g \in\ L\,(R_+)$ or $f,\, g \in\ L^2\,(R_+)$ we define their dyadic convolution $f * g$ by the equality

$$(f * g)\ (x) \quad = \quad \int_{R_+} f\,(x \oplus y)\ g\,(y)\ dy\,, \quad x \in R_+.$$

Theorem 1. *Let the function $K_1\ \in\ L\,(R_+)$ is such that $\inf\limits_{0 \le x < 2^n} \left|\tilde{K}_1\,(x)\right| > 0$ for every $n \in Z_+$. Then if the limit*

$$\lim_{x \to +\infty} \quad \int_{R_+} K\,(x \oplus y)\ f\,(y)\ dy \quad = \quad C\,(f) \int_{R_+} K\,(y)\ dy$$

exists for some function $f \in L^\infty\,(R_+)$ and $K = K_1$, then this limit exists for all functions $K \in L\,(R_+)$.

Theorem 2. *Let f be a function of the space $L\,(R_+)$. Then the linear hull of the set $\{f\,(\cdot \oplus y)\ :\ y \in R_+\}$ is dense in the space $L\,(R_+)$, iff $\inf\limits_{x \in [0,a]} \left|\tilde{f}\,(x)\right| > 0$ for all $a > 0$.*

Theorem 3. *Let f be a function of the space $L^2\,(R_+)$. Then the linear hull of the set $\{f\,(\cdot \oplus y)\ :\ y \in R_+\}$ is dense in the space $L^2\,(R_+)$, iff $\tilde{f}\,(x)\ \neq\ 0$ for almost all $x \in R_+$.*

Theorem 4. *Let be given a function $f \in L\,[0,1)$ or $f \in L^2\,[0,1)$. Then the linear hull of the set $\{f\,(\cdot \oplus y)\ :\ y \in [0,1)\}$ is dense in the space $L\,[0,1)$ (correspondingly in the space $L^2\,[0,1)$), iff $\hat{f}\,(n) \equiv \int_0^1 f\,(x)\ w_n\,(x)\ dx \neq 0$ for all $n \in Z_+$, where $\{w_n\,(x)\}_{n=0}^\infty$ is orthonormal Walsh system.*

These results are dyadic analogues of the results of N. Wiener (see [2, chapter 2]). The results of N. Wiener were generalized to the spaces $L\,(G)$ and $L^2\,(G)$, where G is locally compact abelian group (see [3]). We consider the group R_+ with the topology generated by the base of the neighborhoods $[0\,,\,2^{-n})_{n \in Z}$ of the zero. This group is noncompact. Therefore our results does not follow from the generalizations mentioned above.

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Recent results on refinable bases on the interval

Laura Gori†, Laura Pezza‡, Elisabetta Santi*

†,‡*University of Roma "La Sapienza", Italy,*
**University of L'Aquila, Italy*

†*gori@dmmm.uniroma1.it*, ‡*pezza@dmmm.uniroma1.it*,
**esanti@ing.univaq.it*

In this communication we shall present the construction of certain bases of refinable functions and wavelets on the interval. Besides, we shall give some applications to particular classes of approximation operators.

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Optimization techniques in designing piece-wise polynomial interpolators of minimal support

Atanas Gotchev[†], Tapio Saramäki[‡], Karen Egiazarian[*]

Tampere University of Technology, Finland

[†]*agotchev@cs.tut.fi*, [‡]*ts@cs.tut.fi*, [*]*karen@cs.tut.fi*

Reconstruction of a certain continuous function using given uniformly sampled data is a common problem in many image processing tasks such as image enlargement, rotation, rescaling, etc. The problem is widely known as image interpolation.

Historically, classical interpolators have been designed to comply with the interpolation constraint, i.e. to go through zero for integer coordinates thus preserving the given discrete samples in the reconstructed continuous function. The designs have been targeting approximation of the ideal (for band-limited functions) sinc function, either in time or frequency domain. Short kernels have been preferred for complexity reasons.

Recently, the use of non-interpolating spline-based reconstruction functions has been favored in the light of modern approximation theory. The approximation order has been adopted as a key index that pre-determines the interpolation quality. What makes the splines preferable for interpolation is their efficiency since they can achieve a desired approximation order L with a minimum support of $L + 1$. A low-cost pre-filtering step should be performed to cope with the non-interpolating behavior of the reconstruction function. Additionally, the $L-$order spline-based interpolators can be optimized to have the smallest possible approximation constant thus sizing the minimal asymptotic approximation error. Functions of those class known as splines of minimal support or *moms* (maximal order minimal support) have been reported as superior for image interpolation.

While the approximation order and the approximation constant are asymptotic accuracy measures, there is a Fourier domain error kernel, well known to the approximation theorists, that characterizes the reconstruction function's behavior over the whole frequency range of interest.

In this work, we adopt the use of uniform B-splines of different degrees as linearly combined bases for forming piece-wise polynomial interpolators and keep the support minimal (efficiency not compromised!), namely equal to the highest degree B-spline's support. The weighting parameters in the combination are tuned by an optimization mechanism inspired by the digital filter design rather than the approximation theory. In our setting we consider the interpolator's frequency response as formed by adjoined regions: pass-band, transition band, and stop-bands. We aim at approximating the ideal band-limited interpolator in the pass-band and at suppressing unwanted frequencies in the stop-band as much as possible. Define the pass-band as a fraction of the Nyquist rate and consider the images of this band around $2k\pi$ as the aliasing frequency components to be attenuated. Forming stop bands means, in terms of filter design, clustering of some zeros around $2k\pi$. In contrast with the $L-$order interpolators where $L-1$ zeros are placed exactly at $2k\pi$, this approach sacrifices the multiplicity, distributing the zeros among the stop-band, hence providing better stop-band attenuation.

While a least squares optimization working directly on the Fourier domain error kernel is achievable, we show that a minimax optimization task specified in Fourier domain leads to very good results both in experiments with real images and in terms of error kernel behavior. In our design the error kernel is not asymptotically optimal but keep very good shape for high frequencies close to the Nyquist rate. For low-frequencies the error is very small and controllable through the optimization procedure.

Fast spline-regularization algorithms for computer tomography

Alexandre Grebennikov

Benemerita Universidad Autonoma de Puebla, Mexico

agrebe@fcfm.buap.mx

Computer Tomography consists in reconstruction the image of an interior of a body using the measurements on its surface of characteristics of some external field. Very often it can be state mathematically as an ill-posed coefficient inverse problem for a differential equation describing the distribution of the field in considered region. Coefficients are the functions of the space variables and characterize properties of a media. Well known are X-rays, ultrasonic, radioisotope, infrared and electrical impedance tomography. For example, X-rays tomography leads to the linear mathematical model and for its resolving can be used corresponding linear algorithms, but the know methods for the electrical impedance tomography imaging are nonlinear [1].

We propose another approach for the mathematical modeling the Computer Tomography. We explain it here for the electric impedance tomography imaging in the plane case. Proposed new model of the traditional measurement scheme leads to the linear method. In the radial symmetric case we obtain equation with the classic Radon transformation. In the general case we introduce a new G-transformation, related with the Radon transformation. We obtain the basic integral-differential equation as a mathematical model. Fast regularization algorithms for its numerical solution, based on the Recursive Spline Smoothing [2], are constructed. These approach and algorithms are justified theoretically and with the numerical experiments on the simulated model examples, related with the application of the electric tomography in industry and medicine. Some mentioned results are presented in [3, 4].

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On projection methods for ill-posed problems with a posteriori choice of the dimension of spline or wavelet subspaces

Uno Hämarik

University of Tartu, Estonia

uno.hamarik@ut.ee

We consider linear problems $Au = f$ with bounded operator A between infinite dimensional Hilbert spaces. It will be assumed that the range of the operator is non-closed, therefore this problem is ill-posed. We assume furthermore that instead of f noisy data f^δ are given satisfying $\|f^\delta - f\| \le \delta$. We discuss projection methods for finding approximation u_n to minimum-norm solution u_* of problem $Au = f$ in assumptions garanteeing in case $f^\delta = f$

convergence $u_n \to u_*$ $(n \to \infty)$. Here n is dimension of projection subspace which may consist from splines or wavelets. If these assumptions hold and only noisy data f^δ are available, it is possible to choose $n = n(\delta)$ so that $u_{(n)\delta)} \to u_*$ for $\delta \to 0$. This phenomenon is called the self-regularization by projection. We consider self-regularization by the following projection methods: the least error method, the least square method, the Galerkin method, special collocation method for integral equations of the first kind. For choice of dimension $n = n(\delta)$ both a priori and a posteriori rules are considered. The main emphasis is devoted to a posteriori rules. We consider discrepancy principle [2,4,5], the monotone error rule [2] and discuss also other a posteriori rules in [1, 3].

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Obtaining wavelets and framelets adapted to differential operators

Oleg N. Ipanov[†], Victor G. Zakharov[‡]

Institute of Continuum Mechanics of RAS, Perm, Russia

[†]*ion98@matmod.pstu.ac.ru*, [‡]*victor@icmm.ru*

Matrices of differential operators in wavelet basis usually have finger-wise structures. In spite of sparse form of the matrices implementation of efficient algorithms of operator action requires fast multiplication of sparse matrix on (sparse) vector. In our viewpoint namely complicated algorithms of sparse matrix-vector multiplication lead to the fact that using of wavelets for differential equations solution is not widely practiced so far. Nevertheless it is possible to obtain a diagonal form of linear differential operator representation using *biorthogonal* wavelet bases. If ψ_{jk} is an orthonormal wavelet basis then biorthogonal wavelet bases with respect to operator L can be obtained as follow

$$\alpha_{jk} = L^{-1}\psi_{jk}, \quad \beta_{jk} = L^{*}\psi_{jk}, \tag{1}$$

where L^{*} is an operator conjugated to L. So solution of differential equation $Lu = f$ has the simple form

$$u = \sum_{j,k} \langle f, \psi_{jk}\rangle \alpha_{jk} = \sum_{j,k} \langle f, \beta_{jk}\rangle \psi_{jk}.$$

Formally this approach can be applied for arbitrary differential operator L. Nevertheless two problems arise when we want to obtain biorthogonal bases using (1). The first problem is that direct application of differential operator to wavelet function is ill-posed procedure especially in case of compactly supported wavelets. The second problem is caused by the wish to know scaling functions corresponding to biorthogonal bases in order to use fast wavelet transform algorithm. We know the solution of these problems, using simple modification of masks of orthogonal wavelets, in case of simplest differential operators $\frac{d^k}{dx^k}$ only. For more general differential operators such as the sum of differential operators with constant coefficients situation is more complicated because of biorthogonal basis functions are now not scale invariant. Thus it is necessary to use so-called second generation wavelets. According to this conception masks of biorthogonal wavelets and scaling functions are different on each scale level.

For several examples of operators in the form of $\sum_{k=0}^{N} a_k \frac{d^k}{dx^k}$ coefficients of biorthogonal wavelet and scaling function masks were numerically found. Also more general approach of masks determination is being developed. Besides biorthogonal bases a method of building wavelet frames adapted to differential operators is being developed. For obtaining tight frames so-called Unitary Extension Principle was used. This principle supplies enough simple method of wavelet frames construction with desired properties.

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On connection between the classical and the dyadic Besov spases[8]

I. P. Irodova
Yaroslavl State University, Russia
Irodov@adm.yar.ru

Let F_n be a partition of the cube $Q_0 = [0,1]^d$ into parallelepipeds determined by a set of $(2^n - 1)d$ hyperplanes. We assume that the edges of the parallelepipeds forming F_n are of the length $\approx 2^{-n}$. If F_{n+1} is a refinement of F_n for all n, then the family $F = \bigcup_{n=0}^{\infty} F_n$ is called dyadic family. Let $P^k(F_n)$, denote the set of functions of the form

$$S = \sum_{Q \in F_n} P_Q \chi_Q,$$

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where P_Q is a polynomial in P_k, and χ_Q is the characteristic function of the cube Q.

Let $S^k(F_n)$ denote the set of splines of degree $k-1$.

We recall that a function $f \in L_p(Q_0)$ belongs to the classical space $B_p^{\lambda\theta}$ if the following quantity is finite:

$$|f|_{B_p^{\lambda\theta}} := (\sum_{n=0}^{\infty}(2^{n\lambda} dist(f, S^k(F_n)))^{\theta})^{\frac{1}{\theta}},$$

here $\lambda < K$.

If $S^k(F_n)$ is replaced by $P^k(F_n)$ then we obtain the definition of the dyadic Besov space.

The dyadic Besov spaces are closely related to the classical spaces. Dyadic spaces are wider than classical, but in [1] it was proved that any classical Besov space can be represented as the intersection of the dyadic space $B_p^{\lambda\theta}(F^j)$ with some especially chosen families F^j $(j = 1, ..., 2^d)$.

Theorem 1. *If* $0 < \lambda < min(1, \frac{1}{p})$, $0 < p < \infty$, *then*

$$B_p^{\lambda\theta} \approx B_p^{\lambda\theta}(F).$$

Theorem 2. *Let* $Nd < \lambda < Nd + min(1, \frac{1}{p})$, *then for some constant* $c = c(\lambda, d, \theta, p)$

$$||f||_{B_p^{\lambda\theta}} \leq c||f||_{B_p^{\lambda\theta}(F)}$$

Here f *belongs to the space* $B_p^{\lambda\theta}$.

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Galerkin-wavelets methods for the Sturm–Liouville problems

K. Ivaz
University of Tabriz, Iran
ivaz@tabrizu.ac.ir

In this paper, we review briefly the construction of wavelet orthonormal bases through multiresolution approximations. The multiresolution structure of wavelet orthonormal bases provides a simple way to adapt computational refinements to the local regularity of the solution. In this paper we explain how wavelets can be used to solve the Sturm–Liouville equation with boundary conditions.

Image deblurring using LPA and wavelets

Vladimir Katkovnik[†], Karen Egiazarian[‡], Jaakko Astola[*]
[†]*Kwangju Institute of Science and Technology, Republic of Korea,*
[‡,*]*Tampere University of Technology, Finland*
[†]*katkov@kjist.ac.kr*, [‡]*karen@cs.tut.fi*, [*]*jta@cs.tut.fi*

We describe a novel approach to noisy image deblurring. The approach is based on the intersection of confidence intervals (ICI) rule [4, 5, 7, 8] and gives the algorithm, which is simple to implement and nearly optimal in the point-wise mean squared error risk. The local polynomial approximation (LPA) of the image object function is applied in order to derive the $2D$ scaled kernel estimators (e.g. [8]) used with the deblurring operators. The ICI rule defines the adaptive point-wise varying scales (window sizes) for these kernel estimators and enables the algorithm to be spatially adaptive in the sense that its quality is close to that which one could achieve if the smoothness of the estimated object function was known in advance. The regularized inverse and Wiener regularized filters serve as the deblurring operators.

Suppose we wish to recover an image $y(x)$, $2D$ image intensity function, but we able to observe only $(y \star v)(x)$, where v is the blurring point spread function (PSF) of a linear discrete convolution. The blurring phenomenon modelled by the kernel v (continuous or discrete) is very evident in many image applications.

Moreover, we assume that the data are noisy, so that we observe $z(x)$ given by $z(x) = (y \star v)(x) + \eta(x)$, where x is defined on the regular $n_1 \times n_2$ lattice $X = \{(k_1, k_2),\, k_1 = 1, 2, ..., n_1,\, k_2 = 1, 2, ..., n_2\}$ and η is a zero mean white Gaussian noise with the variance σ^2. We are interested in recovering the image $y(x)$ from the data $z(x)$ which are distorted by the blurring convolution as well as the additive noise. It is assumed that the PSF v is known.

An unbiased estimate can be obtained as the pure inverse. However, in the cases of most interest, v is not invertible, in the sense that the inverse transform does not exist as a bounded linear operator; such inverse problems are called ill-posed or ill-conditioned. It is now standard to approach the inverse problem by the method of regularization, in which one applies, rather than the inversion, a regularized inverse operator [2]. A special common point of most methods starting from the frequency domain is that some basis functions are applied for a approximation of the object function $y(x)$ in the form of series with coefficients defined from the observations. These functions may be Fourier harmonics, eigenfunctions of the convolution operator in SVD methods or wavelets in wavelet based decompositions (e.g. [1, 10]).

Basically different ideas and methods arise from a pointwise nonparametric estimation approach [3, 6]. These methods mostly do not assume any underlying global parametric model of the object and do not use the global parametric series for object approximation. It is assumed only that the object is composed from piecewise smooth elements and every point of the object allows a good local approximation. The main goal of estimation is to build a pointwise approximation using the observations from a neighborhood. The estimation is produced in a sliding mode what means that the every point of the object is treated in a special way in order to achieve the best possible performance [8]. Spatial point-wise adaptation is now commonly considered as a crucial element of the nonparametric estimation. These adaptation methods even for an originally linear method are finalized in nonlinear estimators [9, 5, 11].

In this paper we employ and extend these results to the problem of noisy image deblurring. Comparison of the LPA-ICI deblurring method with its wavelet-based counterpart shows that images restored by the first method have better visual quality and, as a result, higher universal image quality index [12] than those using wavelet based technique. New two stage scheme where LPA-ICI estimates are used as a pilot estimates in the wavelet-Wiener regularized deblurring scheme is suggested. It provides results better than those reported in literature.

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Image compression approach aimed at reducing the Hausdorff distance

Roman Kazinnik

Tel-Aviv University, Israel

romank@post.tau.ac.il

In this “work-in-progress” presentation we will explain a new approach to image compression, based on finding and computing a compact representation of the singularities of images. Other latest developments in this direction will be overviewed, as well as the basic ideas of our approach. We will demonstrate by few examples of piecewise constant images, that the compressed images obtained by our method “look better” than images obtained by a similar compression method which aims at reducing the L_2 error, and have a smaller L_2–Hausdorff distance. For the class of piecewise constant images, our method outperforms the standart wavelet based Jpeg2000 format.

We plan to extend our approach to the class of natural images, and to characterize theoretically the class of images for which our method is efficient.

This is a joint work with Shai Dekel and Nira Dyn.

Mean values interpolating quadratic splines on simple triangulations

J. Kobza

UP Olomouc, Czech Republic

kobza@inf.upol.cz

There are many results known in the theory of function and derivatives values interpolation with splines of satisfactory high degree in 2D (see an overview by Nuernberger–Zeilfelder in J. Comput. Appl. Math. 2000. Vol. 121. P. 125–152; bibliography on FEM). The technique of the BB control net is frequently used in the description of proper sets of points of interpolation, in formulation of continuity conditions and construction of interpolants. Problems with the dimension of interpolating spline spaces were described and solved in many articles and several books. Some results concernig mean values interpolation in 2D we can find in several articles and in the monograph by Bojanov–Hakopian–Sahakian.

In this contribution we shall use the various local representations of quadratic spline on the triangle with function and mean

values as local parameters. We will formulate the mean value interpolation (MVI) and continuity conditions as relations between such parameters and use them to give the answer to the following problem:
Let us have given some simple triangulation with prescribed two-dimensional mean values for some subset of triangles. Does there exist quadratic spline which interpolates these values prescribed?

We cannot expect the existence of such a solution with mean values prescribed for aech triangle on very extensive triangulations. On small triangulations there will be some free parameters, which can be chosen or used for some optimization purposes. The problem of "Lagrangian set" of triangles with prescribed mean values or another parameters appears on most triangulations.

We will formulate the results concerning
– MVI quadratic spline local representations and continuity conditions,
– MVI spline on some simple triangulations (two triangles, three - six triangles, $\Delta^1(m,n), \Delta^2(m,n)$ - triangulations),
– examples of MVI problems with 1D mean values prescribed along edges of triangles.

Ahmed–Rao bases and wavelet expansions[9]

A. V. Korovkin[†], V. N. Malozemov[‡], S. M. Masharsky[*]

St. Petersburg State University, Russia

[†]*korovkin@ipc-spb.ru*, [‡]*malv@gamma.math.spbu.ru*, [*]*smash@scientist.com*

Given $N = 2^s$, we explicitly construct a collection of s orthogonal bases in a space of complex-valued N-periodic functions of integer argument. These bases are of the following property: elements of

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the r-th basis (where $r \in \{1, \ldots, s\}$) take 2^r values equally spaced on a unit circle of complex plane. We show that each basis is a final one in some recurrent sequence of orthogonal bases having block structure. From blocks belonging to *different bases* of a recurrent sequence we form generalized wavelet bases constituting a wavelet packet.

For all considered bases we obtain effective formulas of decomposition and reconstruction.

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Wavelet-based algorithms for pattern recognition

Inna Kozlov

Israel Institute of Technology, Haifa, Israel
eorddik@techunix.technion.ac.il

In the time being, wavelet analysis becomes a steadily fast growing research field with huge numbers of applications.

It, in particular, has been employed in signal and image processing, communication systems, biomedical imaging, radar, acoustics, geophysics and so on.

The goal of this lecture is to demonstrate new application oriented wavelet technique in the area of pattern recognition.

Zhamalov–Weiss distribution: Battle–Lemarie wavelets revisited II

T. E. Krenkel[†], E. T. Krenkel[†], K. O. Egiazarian[‡], J. T. Astola[*]

[†]*Moscow Technical University for Communications and Informatics, Russia,*
[‡,*]*Tampere University of Technology, Finland*
[†]*krenkel2001@mail.ru*, [‡]*karen@cs.tut.fi*, [*]*jta@cs.tut.fi*

Wavelets are objects on the real line which are stable under the action of an affine group (dilations and translations). In the Fourier domain they can be designed with the aid of the Sinnott–Kubert distributions or universal ordinary distributions on $\mathbb{Q}/\mathbb{Z}$ with values in an torsion-free abelian group, satisfying (for every positive integer f) the following relation (Kubert identity):

$$\phi(fx) = f^{s-1} \sum_{j=0}^{f-1} \phi(x + f^{-1}j).$$

For each complex constant s, there are exactly two linearly independent continuous functions, defined on the open interval $(0,1)$, which satisfy Kubert identity. The two generators may be chosen so that one is even and one is odd under the involution $\phi(x) \mapsto \phi(1-x)$. Using the periodic polylogarithms of the negative integer order, we demonstrate how orthonormalized wavelets of Battle–Lemarie can be designed.

On spherical partial sums of multiple trigonometric series

O. I. Kuznetsova

Institute for Applied Mathematics and Mechanics NAS, Ukraine

kuznets@iamm.ac.donetsk.ua

О сферических частичных суммах кратных тригонометрических рядов

Предполагается рассмотреть некоторые аспекты поведения сферических частичных сумм. При любом $p \geq 1$ найден точный порядок роста норм p-сильных сферических средних рядов Фурье измеримых ограниченных почти всюду на m-мерном торе $T^m (m \geq 3)$ функций. При $m = 2$ оценка сверху была получена автором ранее [1, 2]. Доказана неулучшаемая оценка линейных средних сферических ядер Дирихле в интегральной метрике. Как следствие, для кратных тригонометрических рядов с радиальными коэффициентами получены условия, при выполнении которых рассматриваемый ряд есть ряд Фурье, и необходимые и достаточные условия сходимости этих рядов по сферам в среднем. В частности, для рядов

$$\sum_{k \in Z^m} \frac{b(|k|)}{|k|^{\alpha}} e^{ikx},$$

где $\alpha > 0$, $b(t)$ — медленно меняющаяся (по Зигмунду) функция, доказан критерий сходимости по сферам в $L(T^m)$.

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On constructing polyharmonic tension splines

Boris I. Kvasov

Institute of Computational Technologies RAS, Russia

kvasov@lchd.ict.nsc.ru

This paper addresses a new approach in solving the problem of multidimensional spline interpolation. Based on the formulation of the latter problem as a differential multipoint boundary value problem for polyharmonic tension spline we consider its finite-difference approximation. The resulting system of linear equations can be efficiently solved by successive over-relaxation iterative method or using finite-difference schemes in fractional steps. We consider the basic computational aspects and illustrate the main advantages of this original approach.

Color Wavelet–Haar–Prometheus transforms for image processing

Ekaterina L.-Rundblad, Alexei Maidan[†], Valeri Labounets[‡]

Urals State Technical University, Ekaterinburg, Russia

[†]*alexei_maidan@front.ru*, [‡]*lab@rtf.ustu.ru*

This paper present a new approach to the Color Fourier Transformations. Color image processing is investigated in this paper using an algebraic approach based on triplet (color) numbers.

The so-called orthounitary transforms are introduced and used for color image processing. These transforms are similar to a fast orthogonal and unitary transforms. Simulations on use of the color Wavelet–Haar–Prometheus transforms on color image compression have also been performed.

Our approach to color image processing is in using so-called color triplet numbers for color images and to operate directly on three-channel (RGB-valued) images as on single-channel triplet-valued images.

In our approach, each image color pixel is considered not as a 3D RGB vector, but as a triplet number in the following two forms: $\mathbf{f}_{col}(\mathbf{x}) = f_R(\mathbf{x})1_{col} + f_G(\mathbf{x})\varepsilon_{col} + f_B(\mathbf{x})\varepsilon^2_{col}$, $\mathbf{f}_{col}(\mathbf{x}) = f_{lu}(\mathbf{x})\mathbf{e}_{lu} + \mathbf{f}_{Ch}(\mathbf{x})\mathbf{E}_{Ch}$, where $\varepsilon^3 = 1$. The numbers of the form $\mathcal{C} = x + y\varepsilon_{col} + z\varepsilon^2_{col} =$ are called the *triplet* or *color numbers.* Every color number $\mathcal{C} = x + y\varepsilon^1 + z\varepsilon^2$ is a linear combination $\mathcal{C} = x+y\varepsilon^1+z\varepsilon^2 = a_{lu}\mathbf{e}_{lu}+\mathbf{z}_{Ch}\mathbf{E}_{Ch} = (a_{lu}, \mathbf{z}_{Ch})$ of the “scalar” $a_{lu}\mathbf{e}_{lu}$ and “complex” parts $\mathbf{z}_{Ch}\mathbf{E}_{Ch}$ in the idempotent basis $\{\mathbf{e}_{lu}, \mathbf{E}_{Ch}\}$. Thus $f_{lu}(\mathbf{x})$ is a real-valued (grey-level) image and $\mathbf{f}_{Ch}$ is a complex-valued (chromatic-valued) image.

2D discrete color image can be defined as a 2D array $\mathbf{f}_{col} := [\mathbf{f}_{col}(i,j)]^N_{i,j=1}$ i.e., as 2D discrete $\mathcal{A}^{col}_3$-valued function in one of the following forms $\mathbf{f}_{col}(i,j) : \mathbf{Z}^2_N \to \mathcal{A}^{col}_3(1, \varepsilon^1, \varepsilon^2)$, $\mathbf{f}_{col}(i,j) : \mathbf{Z}^2_N \to \mathcal{A}^{col}_3(\mathbf{R}, \mathbf{C})$, $\quad (i,j) \in \mathbf{Z}^2_N$.

The set of all such images forms N^2D Greaves–Hilbert space $\mathbb{L}(\mathbf{Z}_N^2, \mathcal{A}_3^{col}) = \left(\mathcal{A}_3^{col}\right)^{N^2} = \mathbf{R}^{N^2}1 + \mathbb{R}^{N^2}\varepsilon^1 + \mathbf{R}^{N^2}\varepsilon^2 = \mathbf{R}^{N^2}\mathbf{e}_{lu} + \mathbf{C}^{N^2}\mathbf{E}_{Ch} = \mathbf{R}^{N^2} \oplus \mathbf{C}^{N^2}$, where $\mathbf{R}^{N^2}1$, $\mathbf{R}^{N^2}\varepsilon^1$, $\mathbb{R}^{N^2}\varepsilon^2$ are real N^2 Hilbert spaces of red, green, and blue images, respectively, and $\mathbf{R}^{N^2}$ is N^2D real space of gray-level images, and $\mathbf{C}^{N^2}$ is N^2 complex spaces of chromaticity images.

A color linear operator $\mathfrak{L}_{2D} : \left(\mathcal{A}_3^{col}\right)^{N^2} \longrightarrow \left(\mathcal{A}_3^{col}\right)^{N^2}$, $\mathfrak{L}_{2D}[\mathbf{f}_{col}] = \mathbf{F}_{col}$ is said to be *orthounitary* if $\mathfrak{L}_{2D}^{-1} = \mathfrak{L}_{2D}^*$. Orthounitary operators preserve scalar product $\langle .|.\rangle$ and form orthounitary group transforms $\mathbb{OU}(\mathcal{A}_3^{col})$. This group is isomorphic to direct sum of orthogonal and unitary groups $\mathbb{O}(\mathbb{R})\mathbf{e}_{lu} + \mathbb{U}(\mathbb{C})\mathbf{E}_{Ch}$ and every its element has the following representation $\mathfrak{L}_{2D} = \mathbf{O}_{2D}\mathbf{e}_{lu} + \mathcal{U}_{2D}\mathbf{E}_{ch}$, where $\mathbf{O}_{2D} \in \mathbb{O}(\mathbf{R})$ and $\mathcal{U}_{2D} \in \mathbb{U}(\mathbf{C})$ are orthogonal and unitary transforms, respectively. For color image processing we have defined separable $2D$ orthunitary transforms.

One of the aims of this paper is to define three-channel transforms (the so-called color Fourier transforms) that could eventually be used in digital color image compression. We have experimented with the so-called *color Wavelet–Haar* $\mathbf{WH}_{2^n}$, *color Wavelet–Haar–Prometheus* $\mathbf{WHP}_{2^n}$, *Wavelet–Haar–Hartley* $\mathfrak{WH}_{3^n}$, and *Wavelet–Haar–Hartley–Prometheus* $\mathfrak{WHP}_{3^n}$, *transforms.*

We have performed a number of simulations on the use of orthounitary transforms in area of color image processing. We have experimented both with $2^n \times 2^n$- and $3^n \times 3^n$-pixel images by using $\mathfrak{WHP}_{2^n}$, $\mathfrak{WHP}_{3^n}$, and $\mathbf{WHP}_{2^n}$, $\mathbf{WHP}_{3^n}$ transforms.

The analysis presented provides a very general framework for the definition of other multicolor transforms based on multiplet hypercomplex numbers. The derivation of such multicolor transforms is the subject of ongoing research. The motivation of this ongoing research is to define multicolor transforms that can be used efficiently in multicolor satellite image compression.

Wavelet based seasonal adjustment and business cycles

M. Laaksonen

University of Vaasa, Finland

matti.laaksonen@uwasa.fi

When analyzing economic data we usually prefer seasonally adjusted time series. Such adjusted data is easily available in national statistics. Widely used methods are X11, X12 and Seats–Tramo which are ARIMA type filtering methods.

In this paper we express an easy adjustment method based on discrete wavelet transform. The method is easy to implement and its flexibility leads to some usefull variants.

We also introduce an Hodrick–Prescott type detrending operator. The joint implementation of cycle estimation and seasonal adjustment is also discussed.

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Construction of univariate and bivariate orthonormal trigonometric splines

Xiaoyan Liu

University of la Verne, USA
liux@ulv.edu

In this paper, the integral iteration formulas for construction of univariate and bivariate trigonometric splines with compact supports are investigated further. The properties of trigonometric splines are explored and applications are discussed. More bivariate orthonormal trigonometric splines on small compact supports are created. The integral formulas can also be used to construct high dimensional trigonometric splines, which are not cross product of univariate trigonometric splines and have their own advantages.

On best approximation of classes by radial functions

V. Maiorov

Israel Institute of Technology, Haifa, Israel
e-mail: maiorov@tx.technion.ac.il

We investigate the radial manifolds $\mathcal{R}_n$ generated by linear combination of n radial functions on $\mathbf{R}^d$. We consider the best approximation of function classes by the manifold $\mathcal{R}_n$. In particular, we prove that the deviation of the manifold $\mathcal{R}_n$ from the Sobolev class $W_2^{r,d}$ in the Hilbert space L_2 behaves asymptotically as $n^{-\frac{r}{d-1}}$. We show the connection between the manifold $\mathcal{R}_n$ and the space of algebraic polynomials $\mathcal{P}_{d,s}$ of degree s. Namely, we prove there exist constants c_1 and c_2 such that the space $\mathcal{P}_{d,s}$ belongs or not belongs to $\mathcal{R}_n$ as $n \geq c_1 s^{d-1}$ or $n < c_2 s^{d-1}$, respectively.

Approximation with relative error of functions accepting zero value

P. S. Malachivskii

Center of Mathematical Modelling NAS, Lviv, Ukraine

psmal@cmm.lviv.ua

Аппроксимация с наименьшей относительной погрешностью функций, принимающих нулевое значение

The problem is considered on uniform (Chebyshev) approximation with least relative error of functions accepting zero value. Properties of such approximation have been investigated, existing theorem has been stated and the corresponding alternance property has been formulated. The possibility of using Remes scheme justified.

Задача аппроксимации с относительной погрешностью функции, которая на интервале приближения принимает нулевое значение некорректная. Если приближаемая функция $f(x)$ в некоторой точке x_0 интервала приближения $[\alpha, \beta]$ принимает нулевое значение, то относительная погрешность её приближения в точке x_0 не определена. Поэтому к рассмотрению этой задачи подойдем следующим образом. Предположим, что возможно найти такое приближение значений функции $f(x)$ выражением $F(a; x)$, которое в точке x_0 совпадает со значением $f(x_0)$, тоесть равное нулю $F(a; x_0) = 0$. В этом случае относительную погрешность аппроксимации функции $f(x)$ выражением $F(a; x)$ в точке x_0 примем равной нулю. При таких обстоятельствах относительная погрешность аппроксимации

$$\delta(x) = \begin{cases} \left(f(x) - F(a;x)\right)/f(x), & \text{если} \quad x \neq x_0 \\ 0, & \text{если} \quad x = x_0 \end{cases} \tag{1}$$

выражением $F(a;x)$ функции $f(x)$, принимающей нулевое значение только в одной точке x_0 интервала приближения $[\alpha,\beta]$, будет непрерывной функцией на $[\alpha,\beta]$. Таким образом, решение первоначальной задачи сводится к нахождению равномерной аппроксимации функции $f(x)$ выражением $F(a;x)$ с наименьшей относительной погрешностью на сегменте $[\alpha,\beta]$ и условием его равности нулю в точке x_0 — $F(a;x_0)=0$.

Теорема. *Пусть непрерывная и строго монотонная на отрезке $[\alpha,\beta]$ функция $f(x)$ только в одной точке x_0 этого отрезка принимает нулевое значение $f(x_0)=0$, а выражение $F_m(a;x)$ зависит от m дейсвительных параметров a и удовлетворяет условию Хаара [1] на $[\alpha,\beta]$. Тогда равномерное приближение функции $f(x)$ выражеием $F_m(a;x)$ с наименьшей относительной погрешностью на отрезке $[\alpha,\beta]$ и условием*

$$F_m(a;x_0)=f(x_0)=0 \tag{2}$$

существует и единственное. Параметры $a(a_i, i=\overline{1,m})$ такого наближения удовлетворяют системе уравнений

$$\begin{cases} F_m(a;x_0)=0 \\ \left(f(z_i)-F_m(a;z_i)\right)/f(z_i)=(-1)^i\mu, \quad i=\overline{1,m}, \end{cases} \tag{3}$$

где μ — значение погрешности аппроксимации, а z_i, $i=\overline{1,m}$ точки альтернанса.

Полученные результаты имеют место и при равномерном приближении выражением $F_m(a;x)$ з наименьшей относительной погрешностью непрерырвной функции $f(x)$, которая в k, $(k<m)$ разных точках $x_{0,i}$, $i=\overline{1,k}$ отрезка $[\alpha,\beta]$ принимает нулевые значения. В этом случае существует единственное равномерное приближение функции $f(x)$ выражением $F_m(a;x)$ з наименьшей относительной погрешностью, которое в точках $x_{0,i}$, $i=\overline{1,k}$, принимает нулевое значение $F_m(a;x_{0,i})=0$.

Значения параметров равномерного приближения с наименьшей относительной погрешностью выражением $F_m(a;x)$ функции $f(x)$, принимающей нулевое значение, можно определить с использованием итерационной схемы Ремеза [1]. Установленное теоремой альтернансное свойство позволяет находить равномерные приближения для непрерывных функций $f(x)$ и в случае их табличного задания.

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Quasi-interpolants on Powell–Sabin partitions

Carla Manni[†], Paul Sablonnière[‡]

[†] *Università di Roma "Tor Vergata", Italy,*
[‡] *INSA, Centre de Mathématiques, Rennes Cédex, France*

[†] *manni@mat.uniroma2.it*, [‡] *paul.sablonniere@insa-rennes.fr*

It is well known that quasi-interpolation is a general approach to construct, with low computational cost, efficient local approximants to a given set of data or a given function.

Quasi-interpolants are usually obtained as linear combinations of B-splines which are suitable positive, compactly supported basis functions in the approximating space.

As far as bivariate splines are concerned, interesting quasi-interpolating schemes have been proposed for tensor-product structures and for subspaces of splines over special three directional and four directional partitions. On the other hand, the case of spline spaces defined over arbitrary triangulations seems unexplored from this point of view.

In this talk we consider the space of quadratic splines of class C^1 over a Powell–Sabin refinement of a general triangulation of a planar domain. Following [1], we discuss the construction of B-splines in this space which are particularly adapted to quasi-interpolation; we present some quasi-interpolating schemes based on them and we analyze their approximation properties.

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Non-separate wavelets for two channel system

I. E. Maximenko

St. Petersburg State University, Russia

irene@ir4558.spb.edu

We consider scaling equations with a matrix dilation M and $|\det M| = 2$, and study the corresponding biorthonormal compactly supported wavelets. Explicit formulas for the wavelets functions are given.

Under the condition that the Fourier transforms of generating scaling functions φ, $\tilde{\varphi}$ decay faster than $1/\sqrt{|u|}$, the shifts and dilations of the corresponding wavelet functions constitute Riesz bases for $L^2(\mathbb{R}^d)$. In particular any function $f \in L^2(\mathbb{R}^d)$ can be expensed to the unconditionaly converged series

$$f = \sum_{j,k} \langle\, f, \psi_{jk} \,\rangle\, \tilde{\psi}_{jk} = \sum_{j,k} \langle\, f, \tilde{\psi}_{jk} \,\rangle\, \psi_{jk}.$$

The concrete example is given.

Analysis of data at scattered sites on the sphere

H. N. Mhaskar

California State University, Los Angeles, U.S.A.

http://www.calstatela.edu/faculty/hmhaska

Many applications in geomathematics require the analysis of "scattered data" of the form $(\mathbf{x}_k, f(\mathbf{x}_k))$ where $\mathbf{x}_k$ are arbitrary points on the sphere, and f is a function on the sphere. We discuss two questions in this direction. The first is to represent f when it is a restriction of a band limited signal to the sphere using finitely many *bits* (rather than real numbers), based on scattered data. The other is to construct a frame consisting of *zonal function networks* to locate the singularities of the function across arcs on the sphere. We do not require that the points $\mathbf{x}_k$ be chosen in any specific manner, for example, to minimize an energy expression. An important tool in both these applications is a numerical integration formula, exact for high degree spherical polynomials.

Quadratic histosplines: algorithms and exact error bounds

V. L. Miroshnichenko

Sobolev Institute of Mathematics, Novosibirsk, Russia

miroshn@math.nsc.ru

Let $\Delta = \{a = x_0 < x_1 < \ldots < x_N = b\}$ be a partition of the interval $[a, b]$ and let the values

$$\sigma_i = \frac{1}{h_i} \int\limits_{x_i}^{x_{i+1}} f(v)dv \ , \qquad i = 0, ..., N-1,$$

are given, where $h_i = x_{i+1} - x_i$ and $f(x) \in C[a, b]$.

An interpolating quadratic histospline is a function $S(x) \in C^1[a, b]$ such that

1. on every interval $[x_i, x_{i+1}]$ the function $S(x)$ is a quadratic polinomial;

2. the interpolation conditions are satisfied

$$\frac{1}{h_i} \int\limits_{x_i}^{x_{i+1}} S(v)\,dv = \sigma_i \ , \qquad i = 0, 1, \ldots, N-1.$$

For the uniqueness of interpolating histospline we assume that the two boundary conditions for $S(x)$ are satisfied. We consider the following types of boundary conditions: (Type I) $S(x_k) = f(x_k), k = 0, N$; (Type II) $S'(x_k) = f'(x_k), k = 0, N$; (Type III) $S^{(r)}(a) = S^{(r)}(b), r = 0, 1$; in this case we assume that $f(x)$ is $(b-a)$-periodic; (Type IV) $S''(x_k + 0) = f''(x_k - 0), k = 1, N-1$.

In this report we discuss various algorithms for the construction of quadratic histosplines and state some results on error bounds for arbitrary and uniform partitions.

Theorem. *Let the quadratic histospline $S(x)$ interpolates the function $f(x) \in W^3[a, b]$ and boundary conditions of type I, II or III are satisfied. Then the estimate*

$$\|S(x) - f(x)\|_C \leq \frac{H^3}{24}\|f'''\|_\infty, \quad H = \max_i h_i,$$

holds, where 1/24 is the best possible constant.

We consider also the problem of optimal boundary conditions and give some numerical examles.

Biorthogonal compactly supported wavelets

I. Ya. Novikov

Voronezh State University, Russia

novikov@func.vsu.ru

Stationary and nonstationary biorthogonal compactly supported wavelets are considered. An example of stationary biorthogonal compactly supported wavelets preserving localization with the growth of smoothness is presented. Two examples of infinitely differentiable nonstationary biorthogonal compactly supported wavelets are examined.

Wavelet expansions with uncorrelated scaling coefficients

L. Novikov[†], M. Skopina[‡]

[†]*Institute for Analytical Instrumentation, RAS, Russia,*
[‡]*St. Petersburg State University, Russia*

[†]*nov@iai.rssi.ru*, [‡]*skopina@sk.usr.lgu.spb.su*

The literature on wavelet theory offers a large set of scaling functions and wavelets, which leads to the need for a proper choice of basis functions based on a certain criterion. To provide DWT with additional properties coordinated with the problem to be solved, we suggest a method for synthesis of wavelets taking into account a priori information on the covariance function of a noise component. We have found that scaling coefficients for a biorthogonal wavelet system can be uncorrelated under the following condition: the covariance function of convolution type $K(x-y)$ for a random signal is a refinable function. Unfortunately, it is impossible to provide uncorrelated wavelet coefficients.

We present an algorithm for construction of such biorthogonal wavelets on the basis of known masks for orthogonal wavelet systems. Implementations are given for two concrete covariance functions which are "quasi-refinable" (refinement equations hold for these functions numerically).

On description of interpolation orbits

V. I. Ovchinnikov

Voronezh State University, Russia

vio@func.vsu.ru

We consider the description of interpolation orbits with the help of new interpolation functor generalizing the Lions–Peetre method of means.

We show that this functor can be used for description of orbits of arbitrary elements with respect to linear operators mapping different couples of weighted L_p spaces, as well as orbits of arbitrary elements with respect to linear operators from the Neumann–Schatten classes mapping Hilbert couples.

Orbital properties of this interpolation functor imply some new embedding theorems for generalized Lions–Peetre spaces.

These results are partially published in [1].

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Spline approximations to integral equations with weakly singular kernels[10]

Arvet Pedas

University of Tartu, Estonia

arvet.pedas@ut.ee

We consider the linear integral equation

$$u(t) = \int_0^b K(t,s)u(s)ds + f(t)\,, \qquad 0 \le t \le b\,. \tag{1}$$

We assume that $f \in C^{m,\nu}(0,b)$, $K \in W^{m,\nu}(\Delta_b)$, $m \in \mathbb{N} \equiv \{1,2,\ldots\}$, $\nu \in \mathbb{R} \equiv (-\infty,\infty)$, $\nu < 1$. Here $C^{m,\nu}(0,b)$, $m \in \mathbb{N}$, $\nu < 1$, is defined as the collection of continuous functions $f : [0,b] \to \mathbb{R}$ which are m times continuously differentiable in $(0,b)$ and satisfy

$$\left|f^{(k)}(t)\right| \le \text{const} \begin{cases} 1\,, & k < 1-\nu\,, \\ 1 + |\log \varrho(t)|\,, & k = 1-\nu\,, \\ \varrho(t)^{1-\nu-k}\,, & k > 1-\nu\,, \end{cases}$$

[10] This work was supported by the Estonian Science Foundation (Research Grant Nr. 4410).

where $0 < t < b$, $\varrho(t) = \min\{t, b-t\}$, $k = 0, 1, \ldots, m$. The set $W^{m,\nu}(\Delta_b)$, with $m \in \mathbb{N}$, $\nu < 1$, $\Delta_b = \{(t,s) \in \mathbb{R}^2 : t, s \in [0,b], t \neq s\}$, consists of m times continuously differentiable functions $K : \Delta_b \to \mathbb{R}$ satisfying

$$\left|\left(\frac{\partial}{\partial t}\right)^i \left(\frac{\partial}{\partial t}+\frac{\partial}{\partial s}\right)^j K(t,s)\right| \le \text{const} \begin{cases} 1\,, & \nu + i < 0\,, \\ 1 + |\log|t-s||\,, & \nu + i = 0\,, \\ |t-s|^{-\nu-i}\,, & \nu + i > 0\,, \end{cases}$$

with all non-negative integers i and j such that $i + j \le m$.

It follows from $f \in C^{m,\nu}(0,b)$, $K \in W^{m,\nu}(\Delta_b)$ that if equation (1) has an integrable solution $u \in L^1(0,b)$ then $u \in C^{m,\nu}(0,b)$ (cf. [1]). This complicates the construction of numerical solutions with high accuracy for (1). In the talk a discussion about the attainable order of convergence of spline collocation methods for (1) with numerical illustration would be given.

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Riesz wavelet bases in the space $\ell^2(\mathbb{Z})$

A. B. Pevnyi

Syktyvkar State University, Russia

pevnyi@syktsu.ru

A nonstationary multiresolution analysis $\{V_k\}_{k \ge 0}$ in the space $\ell^2(\mathbb{Z})$ is constructed. The subspaces V_k consist of discrete splines.

The space V_k is the same as the space of discrete splines $\mathbf{S}_{p,2^k}$ of the order p with distance 2^k between the knots.

In every V_k there is a function φ_k such that the system $\{\varphi_k(\cdot - l2^k) : \ l \in \mathbb{Z}\}$ forms Riesz base of V_k. We have $V_k \subset V_{k-1}$ and define wavelet space W_k as orthogonal complement of V_k in V_{k-1}. We define and characterize the wavelets ψ_k whose shifts $\psi_{kl}(j) = \psi_k(j - l2^k)$, $l \in \mathbb{Z}$, form Riesz bases of the wavelet space W_k. This wavelets ψ_k are not generated by dilations of the unique function.

Subspaces W_k form a orthogonal expansion of the space: $\ell^2(\mathbb{Z}) = \oplus_{k=1}^{\infty} W_k$. As the final result for every natural p a multiresolution analysis of $\ell^2(\mathbb{Z})$ is obtained (for $p = 1$ Haar multiresolution analysis is obtained).

A construction of wavelet analysis in the space of discrete splines $S(j)$ which may grow as $O(|j|^s$ as $|j| \to \infty$ is developed in [1].

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On some applications of certain new wavelet bases

Laura Pezza

Università di Roma “La Sapienza”, Italy

pezza@dmmm.uniroma1.it

In this communication, a new class of biorthogonal bases on a finite interval are presented together with some applications. The refinable bases are constructed by starting from pairs of dual refinable functions on the real axis introduced in [3], so that to preserve the stability property and the order of polynomial exactness. In this way, the biorthogonal wavelet bases, constructed by a stable completion, are shown to fulfill the vanishing moments property and so they are suitable to be used in many interesting applications [1, 4]. In particular, here they are used to discretize an integral operator, by Galerkin or Petrov–Galerkin type techniques [2]. It is shown that the discretization matrices they give rise to have only a few entries with significative size and then they can be compressed without reducing the approximation power of the operator. The structure of these matrices is studied and the matrix compression factors are computed; the results are compared with the ones obtained using the B-splines.

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Aliasing spectra using wavelet filtering in physics

Gabor Por[†], Imre Pázsit[‡]

[†]*Budapest University of Technology and Economics, Hungary,*
[‡]*Chalmers University of Technology, Göteborg, Sweden*
[†]*por@reak.bme.hu*, [‡]*imre@nephy.chalmers.se*

Fluctuations measured in fusion technique have been evaluated using wavelet decomposition and wavelet filtering method. Wavelet filtering was used as prefilter before estimation of the auto power spectral densities. It was found that wavelet filtering can cause appearance of additional peaks in the spectra, much smaller in amplitude than the original background level. However, such peeks can lead to misinterpretation of physical results.

Effects are demonstrated from one side in actual physical measured data from JET TOKAMAK. On the other hand we demonstrate the aliasing effect, simulating artificial signals, filtering that by wavelet technique and both reconstructing the signal and estimating their spectrum.

The main aim of this presentation is to invoke discussion on the distortions introduced by wavelet filtering.

Refinement equations and corresponding linear operators

Vladimir Protasov

Moscow State University, Russia
vladimir_protassov@yahoo.com

Refinement equations play an exceptional role in the study of wavelets, they are also used in some problems of approximation theory, computer design, probability theory and combinatorial number theory. We restrict ourselves to univariate equations with compactly supported mask:

$$\varphi(x) = \sum_{k=0}^{N} c_k \varphi(2x - k).$$

The polynomial $m(z) = \sum_{k=0}^{N} c_k z^k$ is the *symbol* of the equation. We assume $m(1) = 1$. The matrix approach consists of considering two linear operators T_0, T_1 acting in $\mathbf{R}^N$ and defined by their $N \times N$ matrices as follows: $(T_0)_{ij} = c_{2i-j-1}$, $(T_1)_{ij} = c_{2i-j}$. Denote $B_i = T_i|_V$, $i = 0, 1$, where V is the smallest subspace invariant with respect to both T_0 and T_1, and containing the point $v = \bigl(\varphi(1) - \varphi(0), \ldots, \varphi(N) - \varphi(N-1)\bigr)^{\top}$. This is well known (see, for instance, [1], [2], [3], [4]) that the smoothness of the compactly supported solution φ in the space $\mathbf{C}$ is expressed in terms of the joint spectral radius of the operators B_0, B_1, the smoothness in the space $\mathbf{L}^p$ is in terms of their p-radius, the local regularity is in terms of their spectral radius along sequences. In some important special cases this technique works and gives very sharp information on the regularity of the solutions. In case of general refinement equations the main difficulty is the unclear structure of the subspace V and consequently the form of the operators B_i (see [2]). To overcome this we obtain an explicit characterization of the kernels, root subspaces and all common invariant subspaces of the operators T_0^*, T_1^* in terms of zeros of the symbol m. This allows us to prove the following result:

Theorem. *For any refinement equation both operators B_0, B_1 are nondegenerate; in a suitable basis they have a lower-triangular block form with blocks $\{2^{-1}, \ldots, 2^{-n}, 2^{-n-1}\tilde{T}_i\}$, where the matrices $\tilde{T}_i$, $i = 0, 1$ have no common nontrivial invariant subspace. These operators are constructed by a symbol $\tilde{m}$ of degree $N - n - 1$.*

This leads to several sharp results on global and local regularity of general refinable functions and on the degree of convergence of subdivision schemes.

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Compressing overcomplete representations

Laura Rebollo-Neira

Aston University, Birmingham, United Kingdom

rebollol@aston.ac.uk

The advantages of overcomplete wavelet representations over conventional wavelet basis representations include noise robustness and freedom gained in the selection of the wavelet generating function. However, in practical signal processing applications some disadvantages associated with overcomplete wavelet representations prevent their use. The fact that overcomplete representations require increased storage requirements, for instance, is one of the significant drawbacks. We feel then motivated to ascertain an approach aiming at overcoming the storage problem limitation, while retaining the inherent noise robustness benefit of redundancy. The goal is achieved by means of a transformation specially devised for compressing redundancy.

Given an overcomplete set of functions (typically wavelet or Gabor-like frames) the proposed approach is constructed out of the following ingredients:

a) a criterion and a methodology for designing interpolating kernels;

b) an adaptive biorthogonalization technique giving rise to orthogonal projectors.

The discussion of both such ingredients being the core of the talk.

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On frame coefficients and expansions of L^p functions[11]

T. V. Rodionov

Moscow State University, Russia

rodiontv@mech.math.msu.su, rodionovtv@mail.ru

Let us consider a frame $\Phi = \{\varphi_n\}_{n=1}^{\infty}$ of bounded functions in any measure space X. The aim of this abstract is to present some estimates that relate the L^p norm of function f to sums of certain powers of its frame coefficients $\widehat{f}_n = (f, \varphi_n) = \int_X f(x)\overline{\varphi_n(x)}\, d\mu$ with weights depending only on frame. Our results generalize classical Hausdorff–Young–Riesz and Hardy–Littlewood–Paley theorems as well as other ones on trigonometric series, orthogonal and orthosimilar (orthogonal-like, see [1]) systems by J. Marcinkiewicz and A. Zygmund [2], S. Verblunsky, V. A. Il'in, author [3] et al.

By B denote the upper frame bound of Φ.

Theorem. *Suppose* $0 < M_n = \operatorname{ess\,sup}_{x\in X}|\varphi_n(x)| < \infty$ *as* $n \in \mathbb{N}$. *Then*

1. *for* $p \in (1,2)$ *and* $f \in L^p$ *there exist* $\widehat{f}_n$ *and following estimates are true:* $I_q(\widehat{f}) = \left(\sum |\widehat{f}_n|^q M_n^{2-q}\right)^{1/q} \le B^{1/q}\|f\|_p$, *where* $1/p + 1/q = 1$,
   $J_p(\widehat{f}) = \left(\sum |\widehat{f}_n|^p \delta_n^{p-2} M_n^{p-2}\right)^{1/p} \le \gamma_1(p, B, D)\|f\|_p$, *where positive numbers* δ_n, D *satisfy inequality* $\sum\limits_{n\,:\,\delta_n M_n^2 > z} \delta_n^{-2} M_n^{-2} \le \frac{D}{z}$ *for each* $z > 0$,
   $K_p(\widehat{f}) = \left(\sum |\widehat{f}_n|^2 (\delta_n M_n^2)^{\frac{p-2}{p}}\right)^{1/2} \le \gamma_2(p, B, D)\|f\|_p$;
2. *if for* $p \in (2,\infty)$ *and a sequence* $\mathbf{c} = \{c_n \in \mathbb{C}\}$ *at least one of values* $I_q(\mathbf{c})$, $J_p(\mathbf{c})$, $K_p(\mathbf{c})$ *is finite, then there exist* $f \in L^p$ *such that* $f = \sum c_n \varphi_n$ *in* L^p, *and* $\|f\|_p \le B^{1/p} I_q(\mathbf{c})$,

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$\|f\|_p \le \gamma_1(q,B,D)J_p(\mathbf{c})$, $\|f\|_p \le \gamma_2(q,B,D)K_p(\mathbf{c})$ *respectively. If moreover* $\mathbf{c} \in l^2$, *then* $c_n = \widetilde{f}_n\ \forall n \in \mathbb{N}$, *where* $\widetilde{f}_n = (f, \widetilde{\varphi}_n)$, $\widetilde{\Phi} = \{\widetilde{\varphi}_n\}_{n=1}^{\infty}$ *is the dual frame for* Φ.

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Approximation theory on compact groups

S. F. Rzaev

University of Tabriz, Iran

rzseymur@hotmail.com

It is well known that one of the important ways for studying the relations between constructive and structural properties of functions, is given by Jackson type theorems. In many of papers which are written in this context on manifolds spaces, symmetric spaces rank one and etc. Thus in this paper we consider the generalized shift operator on compact groups and by using this operator we define modulus of smoothness and so we prove central theorems in approximation theory.

Scale equation and subdivision algorithm for quadratic B-splines on non-uniform criss-cross triangulation

Paul Sablonnière

INSA, Centre de Mathématiques, Rennes Cédex, France
Paul.Sablonniere@insa-rennes.fr

Let $\Omega = [a, b] \times [c, d]$ be a bounded rectangular domain and let $X_m = \{a = x_0, x_1, \ldots x_m = b\}$ and $Y_n = \{c = y_0, y_1, \ldots y_n = d\}$ be two non-uniform partitions on $I = [a, b]$ and $J = [c, d]$ respectively. In each rectangle $R_{ij} = I_i \times J_j = [x_{i-1}, x_i] \times [y_{j-1}, y_j]$, for $1 \le i \le m$ and $1 \le j \le n$, we draw the two diagonals and we obtain the so-called *criss-cross triangulation* $\mathcal{T}_{mn}$ of Ω. Let $\mathbb{P}_2^1(\Omega, \mathcal{T}_{mn})$ be the space of C^1 quadratic splines, i.e. of C^1 functions whose restriction to each triangle of $\mathcal{T}_{mn}$ is in the space $\mathbb{P}_2$ of bivariate polynomials of total degree at most 2. We have defined previously a system of $N(m, n) = (m + 2)(n + 2)$ B-splines with supports in Ω generating the space $\mathbb{P}_2^1(\Omega, \mathcal{T}_{mn})$. They are linearly dependent, but any subsystem of $N(m, n) - 1$ B-splines is a basis of the space. In this talk, we consider the subtriangulation $\mathcal{T}_{2m,2n}$ of $\mathcal{T}_{mn}$ obtained by subdividing each rectangle R_{ij} into four subrectangles by drawing in the medians and the two diagonals of each subrectangle. The finer space is generated by $N(2m, 2n)$ small B-splines and the inclusion $\mathbb{P}_2^1(\Omega, \mathcal{T}_{mn}) \in \mathbb{P}_2^1(\Omega, \mathcal{T}_{2m,2n})$ implies that each B-spline of the coarse space is a linear combination of small B-splines: this is the *scale equation* for quadratic B-splines. Then we deduce from this equation the *subdivision algorithm* which allows the computation of B-spline coefficients of a given spline $S \in \mathbb{P}_2^1(\Omega, \mathcal{T}_{mn})$ on the fine mesh in function of its B-spline coefficients on the coarse mesh. We also give a geometrical construction of the new control vertices in function of the control vertices on the coarse mesh. Finally, we give the expressions of the Bernstein–Bezier coefficients of $S \in \mathbb{P}_2^1(\Omega, \mathcal{T}_{mn})$ on each triangle in function of its B-spline coefficients and reciprocally. These algorithms are essential in CAGD for the computation of quadratic spline surfaces and for the construction of multiresolution analyses based on quadratic B-splines and wavelets. The latter point will be developed in a further paper.

Harmonic analysis on totally disconnected groups and irregularities of point distributions

M. M. Skriganov

Steklov Mathematical Institute at St. Petersburg, Russia

skrig@pdmi.ras.ru

We study point distributions in the multi-dimensional unit cube which possess the structure of finite abelian groups with respect to certain p-ary arithmetic operations. Such distributions can be thought of as finite subgroups in a compact totally disconnected group of the Cantor type. We apply the methods of L^q harmonic analysis to estimate very precisely the L^q-discrepancies for such distributions. Following this approach, we explicitly construct point distributions with the minimal order of the L^q-discrepancy for each q, $1 < q < \infty$.

Two-scale equation for final cyclic group[12]

Viktor V. Slavskii

Ugra Research Institute of Information Technologies, Russia

slavsky@uriit.ru

Важную роль в теории вейвлет-анализа играет функциональное уравнение самоподобия

$$\varphi(x) = \sqrt{2} \sum_{k \in M} h_k \varphi(2x - k).$$

[12]This research was supported by the Russian Fund of Basic Investigations (Grants 02-01-01071, 00-15-96165) and the grant center of St. Petersburg University E 02-1.0-120.

Рассмотрим алгебраические свойства этого уравнения, когда $\varphi : G \to R$ — функция определенная на абелевой группе G. Ограничимся случаем, когда $G = Z_N$ — конечная циклическая группа порядка N. Обозначим через $\varphi = [\varphi_0, \varphi_1, \cdots, \varphi_{N-1}]$ — вектор (строку) значений функций φ. Тогда уравнение самоподобия эквивалентно матричному равенству

$$\varphi = \sqrt{2}\varphi \cdot H,$$

где H — квадратная матрица размера $N \times N$, имеющая структуру

$$H = \begin{bmatrix} h_0 & h_2 & h_4 & \cdots & h_{N-2} \\ h_{N-1} & h_1 & h_3 & \cdots & h_{N-3} \\ h_{N-2} & h_0 & h_2 & \cdots & h_{N-4} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ h_1 & h_3 & h_5 & \cdots & h_{N-1} \end{bmatrix} = \left\| h_{(N-i+2j) \bmod \mathrm{N}} \right\|_{i,j=0,\ldots,N-1}$$

Для $N = 2^n$ теория периодических дискретных всплесков изложена в работе [1]. Рассмотрим случай, когда N нечетное число. Для характеристического полинома матрицы H справедливы утверждения:

Теорема 1. *Если простое число N такого, что 2 — первообразный корень в кольце вычетов Z_N, то справедливо разложение*

$$\det(\lambda E - H) = (\lambda - S)(\lambda^{N-1} - D),$$

где $S = h_0 + h_1 + \ldots + h_{N-1}$, D — полином степени $(N-1)$ от коэффициентов $\{h_0, h_1, \ldots, h_{N-1}\}$, $SD = \det H$.

Замечание. Такими простыми числами будут 3, 5, 11, 13, 19, 29, 37, …

Теорема 2. *Если нечетное N делится на 3, то*

$$\det(\lambda E - H) = (\lambda - S)(\lambda^2 - Q)R(\lambda),$$

где $S = h_0 + h_1 + \ldots + h_{N-1}$, Q — полином степени 2 от $\{h_0, h_1, \ldots, h_{N-1}\}$, $R(\lambda)$ — полином степени $(N-3)$ от λ.

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Implementation of the algorithm of nonsymmetric spectrum noise compensation with DSP TMS320C6414

E. A. Smirnova†, M. N. Smirnov‡

St. Petersburg State University, Russia

†east@tepkom.ru, ‡smn@tepkom.ru

Реализация алгоритма компенсации помех с несимметричным спектром на DSP TMS320C6414

На данный момент для защиты от сосредоточенных по спектру помех существуют устройства, основанные на использовании различных фильтров (например, режекторных, адаптивных, и т.п.). Однако, в случае использования режекторных фильтров для подавления узкополосных помех, потребовалось бы столько же фильтров, сколько помех, так как помехи этого типа часто имеют вид серий спектральных линий. Поскольку режекторные и адаптивные фильтры необходимо предварительно настраивать, требуется механизм или оператор, анализирующий специальные признаки полезного сигнала. Таким образом, эти устройства трудно использовать на практике и они экономически не выгодны. Недавно появился новый подход к решению этой проблемы, основанный на симметрии спектров полезного сигнала. При этом, для работы компенсатора, основанного на этом методе, не требуется априорная информация о текущих параметрах помех и, кроме того, его использование не требует изменения структуры приемного канала и не влияет на работу устройств вторичной обработки сигналов. В данной статье рассматривается реализация в реальном режиме времени метода компенсации помех с несимметричным относительно несущей спектром, выполненная на сигнальном процессоре TMS320C6414 в среде разработки Code Composer Studio. В ходе реализации алгоритма компенсации помех, был разработан прибор, не имеющий аналогов в мире, далее называемый компенсатор помех. Кроме того, данный прибор выполняет цифровое детектирование ЧТ, АТ, ОФТ, АМ сигналов. Компенсатор помех выполнен в виде отдельного устройства, подключаемого к различным радиоприемным устройствам, и позволяет решить задачу повышения устойчивости приема в диапазонах КВ, СВ, ДВ, СДВ, СНЧ при действии сосредоточенных по спектру станционных, преднамеренных, индустриальных помех и помех, создаваемых вычислительными средствами, а также существенно упрощает решение задач электромагнитной совместимости.

Interpolation and approximation with splines and related functions

H. Strauss
Universität Erlangen–Nürnberg, Germany
strauss@am.uni-erlangen.de

We first consider Lagrange interpolation by finite dimensional spaces of multivarate spline functions defined on a polyhedral region K in $R^d, d \geq 1$. Sets of points T are characterized such that in every neighborhood of T there exists a configuration of points which admits unique Lagrange interpolation. For this characterization we use locally independent basis functions.

Then best L^2–approximation with respect to semi–norms is investigated. In particular problems with regularization terms are studied. We consider approximation with spline functions but also with conditionally positive definite functions and more general spaces. Results concerning uniqueness and characterization of solutions of the appoximation problems are given. We also consider extremal properties of the solutions.

On a class of spline–type functions

N. A. Strelkov

Yaroslavl State University, Russia

strelkov@uniyar.ac.ru

Let $\mathbb{A}$ be the set of n-dimensional vectors $\gamma = (\gamma_1, \ldots, \gamma_n)$, where $\gamma_1 = 1$ and $\gamma_k = 1$ or $\gamma_k = -1$ for $k = 2, \ldots, n$. Note that $\mathbb{A}$ consists of 2^{n-1} vectors directed along the diagonals of the cube $[-1/2, 1/2]^n$.

Let L be the map of $\mathbb{N}$ to $\mathbb{A}$ such that

$$L: \; s \in \mathbb{N} \;\longrightarrow\; \gamma^{(s)} \in \mathbb{A}. \tag{1}$$

Then (1) defines the sequence

$$\gamma^{(1)}, \gamma^{(2)}, \ldots, \gamma^{(s)}, \ldots \tag{2}$$

For the fixed sequence (2) let

$$\psi^{(L)}(x) = \Big\{ \prod_{s=1}^{\infty} S_{\gamma^{(s)}} \Big\} \chi(x), \tag{3}$$

where

$$S_{\gamma^{(s)}} f(x) = \frac{1}{2} \big\{ f(x + \gamma^{(s)}/2^{s+1}) + f(x - \gamma^{(s)}/2^{s+1}) \big\}$$

and χ is the characteristic function of the cube $[-1/2, 1/2]^n$.

Proposition. *Every function $\psi = \psi^{(L)}$ of the form (3) (with an arbitrary choice of sequence (2)) generates the asymptotically optimal system*

$$\{\psi(x/h - k)\}_{k \in \mathbb{Z}^n}$$

such that the projection-difference approximation of n–dimensional Laplace operator coincides with its simplest difference $(2n+1)$*-point "cross-type" analogue.*

For example if the sequence (2) is stationary, i.e. $\gamma^{(s)} = \gamma \in \mathbb{A}$ for every $s \in \mathbb{N}$, then

$$\psi(x) = \int_{-1/2}^{1/2} \chi(x + t\gamma)\, dt,$$

i.e. in this case $\psi^{(L)}$ is the average of characteristic function χ along the diagonal of unite cube centered at the point x. The corresponding prolongation operator is the known operator of piecewise linear (simplicial) prolongation. In this situation the construction of prolongation operator connects with the simplicial partition of net cells. If the sequence (2) is more complicated, then the behavior of ψ may be irregular. For example, if sequence (2) is the cyclic repeating of 2^{n-1} diagonals of unite cube, then the support of ψ is the set of fractal structure.

Orthogonal wavelet bases in $W_2^m(R)$ and their approximative properties

Yu. N. Subbotin†, N. I. Chernykh‡

Urals Dept. RAS, Ekaterinburg, Russia

†*Yurii.Subbotin@imm.uran.ru*, ‡*Nikolai.Chernykh@imm.uran.ru*

Ортогональные базисы всплесков в $W_2^m(R)$ и их аппроксимативные свойства[13]

В работах [1–2] построены ортогональные базисы всплесков соболевских пространств W_2^m на периоде и на вещественной оси. Здесь, как и в [1], но для случая $W_2^m(R)$ построены базисы всплесков, ортогональные относительно любого заданного скалярного произведения $\langle f,g\rangle = \sum_0^m p_\nu(f^{(\nu)},g^{(\nu)})$ $(p_\nu \ge 0,\ p_0>0,\ p_m=1,\ (u,v)=\int_R u(x)\overline{v(x)}\,dx)$, порождающего одну из эквивалентных норм $\|f\| = (\langle f,f\rangle)^{1/2}$ этого пространства. С этой целью для частного случая L-сплайнов с узлами на сетке $\Delta_j = \{x_s = s/2^j:\ s\in\mathbb{Z}\}$ $(j\in\mathbb{Z})$, порожденных дифференциальным оператором $L_{2m+2k}(\mathcal{D}) = \sum_{\nu=0}^m (-1)^{\nu+k} p_\nu \mathcal{D}^{2\nu+2k}$ $(\mathcal{D}=d/dx,\ k,m\in N)$ указан ряд новых свойств, в частности получено первое интегральное соотношение для интерполяционных L-сплайнов $S_{L,j}(f,x)$ функций $f\in W_2^{m+k}(R)$ и на этот случай обобщено интегральное представление (C. Michelli) фундаментальных L-сплайнов $L_{2m+2k}(x,\Delta_j)$. С помощью этих свойств доказано, что система подпространств

$$V_j = \text{close Lin}\ \{L_{2m+2k}^{(k)}(x-s/2^j;\ \Delta_j):\ s\in\mathbb{Z}\},\ j\in\mathbb{Z},$$

образует (нестационарный) кратномасштабный анализ пространства $W_2^m(R)$, а система функций $\varphi_{j,k}(x)=\varphi_j(x-k/2^j)$, $k\in\mathbb{Z}$,

[13]Работа выполнена при поддержке РФФИ (проекты 02-01-00764, 02-01-00783) и Совета по государственной поддержке ведущих научных школ (проект НШ-1347.2003.1).

определенных преобразованием Фурье $\widehat{\varphi}_j(\omega) = 2^{-j/2}\omega^k \times \times \left(P^2_{2m+2k}(i\omega)\sum 1/P_{2m+2k}(i(\omega+ 2\pi\nu 2^j))\right)^{-1/2} e^{i\left(\sin\frac{k\pi}{2}\right)\omega/2^{j+1}}$, где $P_{2m+2k}(\lambda)$ — характеристический многочлен оператора $L_{2m+2k}(\mathcal{D})$, образует ортонормированный относительно скалярного произведения $\langle\cdot,\cdot\rangle$ базис V_j $(j\in\mathbb{Z})$. Аналогично представляются преобразования Фурье $\widehat{\psi}_j(\omega)$ функций $\psi_j(x)$ порождающих ортонормированные базисы $\psi_{j,k}(x-k/2^j)$ $(k\in\mathbb{Z})$ ортогональных дополнений W_j пространств V_{j+1} до V_j. Оценки скорости сходимости рядов по всплескам $\{\psi_{j,k} : (j,k)\in\mathbb{Z}^2\}$ легко вытекают из следующих оценок приближения функций интерполяционными сплайнами.

Теорема. *При $q\geq 2$ и $0\leq\nu\leq m+k-1$ имеют место для $f(x)\in W_2^{m+k}(R)$ оценки*

$$\|f^{(\nu)}(x)-S^{(\nu)}_{L,j}(f,x)\|_{L_q(R)}\leq\frac{\pi}{2}2^{-(j+1)\left(m+k-l-\frac{1}{2}+\frac{1}{q}\right)}\|f^{(k)}\|_{W_2^m(R)},\ j\in\mathbb{Z},$$

а для $f\in W_2^{2m+2k}(R)$ — оценки

$$\|f^{(\nu)}(x)-S^{(\nu)}_{L,j}(f,x)\|\leq 18\pi(m+k)2^{-(j+1)\left(2m+2k-l+\frac{1}{q}-\frac{1}{2}\right)}\times$$

$$\times\|L_{2m+2k}(\mathcal{D})f\|_{L^2(R)}.$$

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The development of the analog to mathematical package Wavelet Toolbox (MATLAB) using algorithmic language IDL[14]

R. A. Sych

Institute of Solar-Terrestrial Physics, Irkutsk, Russia
sych@iszf.irk.ru

Presented are the results derived from developing the software package for spectral wavelet analysis of astronomical data. The analog is represented by the expansion package of computer mathematics system MATLAB 6.0/6.1 Wavelet Toolbox 2/2.1, which is currently the most powerful tool for studying, creating and applying wavelets, and for carrying out wavelet transforms. The Wavelet Transform Analyzer (WTA) package evolved by our team is designed for employing one- or two-dimensional wavelet transform of observational data. The programs were written by using the data processing and visualization language, IDL (Interactive Dialog Language) which is a default standard for the astronomical community. An added motivation for the use of the IDL language has also been the fact that currently there is no software for processing astronomical data, based on using wavelet transforms. The WTA package provides the following output data: amplitude, power and phase spectra; skeletons or maps of local extrema; local or global energy spectra; measures of local intermittency or contrast; histograms and integral variances; autocorrelation functions; Hilbert transforms; signal reconstruction; one- and two-dimensional cross-correlations of harmonics; decomposition and synthesis of two-dimensional images; and denoising. For the online control of the signal processing there is a possibility of operating with a variety of mother functions (Wave, WHAT, DOG, Morlet, Paul), as well as modifying their frequency and spatial characteristics.

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The study of two-dimensional images of the solar coronal loops with use of wavelet analysis[15]

R. A. Sych

Institute of Solar-Terrestrial Physics, Irkutsk, Russia
sych@iszf.irk.ru

A research project on the study and practical implementation of the Wavelet-analysis method for obtaining spectral characteristics of time series of two-dimensional solar images is presented. Within the context of investigating the oscillatory processes of coronal loops, a spatial-analysis technique for two-dimensional images of EUV emission sources (TRACE, 171 A) is suggested. It is based on identifying two-dimensional sources of quasi-periodic fluctuations, both within the frequency band and at separate harmonics. The use of direct and inverse Wavelet transform as the frequency band filter for each spatial point forms a groundwork for identification. It enables determination (in time and in space) the areas of the image which make the main contribution to the emission at selected frequencies. Dynamic and phase characteristics of two-dimensional variations of sources are obtained. It is shown that the principal source of the variations at high frequencies lies at the loop footpoints. The low-frequency component is basically concentrated within thin tubes delineating magnetic field lines. These magnetic lines are internal structures of the loops. Emission surges of solar material along the loops were observed to occur at some of the harmonics.

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Convergence speed of Fourier series on Y. Meyer's wavelet system in spaces $L_p(\mathbb{R})$

D. B. Tanana

Russian State Professional & Pedagogic University, Ekaterinburg, Russia

dtanana@r66.ru

Скорость сходимости рядов Фурье по системе всплесков И. Мейера в пространствах $L_p(\mathbb{R})$

В статье исследуются аппроксимативные свойства в пространствах $L_p(\mathbb{R})$ при $p \geq 1$ семейства базисов всплесков И. Мейера [2] $\left\{2^{j/2}\psi(2^j x+k)\right\}_{j\in\mathbb{Z},\ k\in\mathbb{Z}}$, детально изученного в случае бесконечной дифференцируемости $\widehat{\varphi}(\xi)$. В дальнейшем считается, что носитель $\widehat{\varphi}(\xi)$ совпадает с отрезком $|\xi|\leq(1+\varepsilon)/2$, $0<\varepsilon\leq 1/3$, и $\widehat{\varphi}(\xi)$ есть функция ограниченной вариации.

Через $W_{\sigma,p}$ обозначим класс целых функций экспоненциального типа σ, след которых на $\mathbb{R}$ принадлежит L_p. Рассматривается вопрос об аппроксимативных свойствах рядов Фурье по системе всплесков И. Мейера в сравнении с приближениями функциями из $W_{\sigma,p}$. Аналогичная задача при гораздо более сильном предположении о бесконечной дифференцируемости $\widehat{\varphi}(\xi)$ решена Новиковым И. Я. [1].

Рассмотрим функцию $F(z) = (f(z)-f(0))/z$. Результат И. Я. Новикова в [1] базируется на доказанной им теореме, что если $F(z)\in W_{\sigma,2}$, $\sigma\leq 2^j\left(\frac{1-\varepsilon}{2}\right)$, то верно следующее равенство: $S_{2^j}(F,x)=F(x)$.

Обобщим это свойство частичной суммы на случай $W_{\sigma,p}$ (при $p\geq 1$) при указанных ослабленных ограничениях на $\widehat{\varphi}(\xi)$.

Теорема. *Пусть $f(z)\in W_{\sigma,p}$ (при $p\geq 1$), $\sigma\leq 2^j\left(\frac{1-\varepsilon}{2}\right)$. Тогда $S_{2^j}(f,x)=f(x)$, где $S_{2^j}(f,x) = 2^{j/2}\sum f_k\varphi(2^j x+k)$, $f_k=\int\limits_{\mathbb{R}} f(t)2^{j/2}\varphi(2^j t+k)\,dt$.*

Приведенное выше утверждение позволяет оценить скорость сходимости ряда Фурье по системе всплесков Мейера в пространстве $L_p(\mathbb{R})$, $p \geq 1$ в сравнении с наилучшим приближением $E_\sigma(f, L_p)$ соответствующей функции целыми функциями экспоненциального типа σ.

Теорема. *Пусть* $f(x) \in L_p(\mathbb{R})$, $p \geq 1$. *Тогда* $\|f - S_N(f, x, \psi)\|_{L_p} \leq cE_{N/6}(f, L_p)$, *где* $N = 2^j + n$, $0 \leq n < 2^j$, c *— константа, зависящая только от* φ. *Здесь* $S_N(f, x, \psi)$ *— частичная сумма ряда Фурье функции* $f(x)$ *по системе* $\{2^{j/2}\psi(2^l x + k)\}_{(l,k)\in\mathbb{Z}^2}$, *содержащая все члены ряда с* $l \leq j - 1$, $k \in \mathbb{Z}$ *и при* $l = j$, $k \leq n$, *где* ψ *— всплеск Мейера, соответствующий любой заданной масштабирующей функции* φ *с указанными свойствами и* $\sum |\widehat{\varphi}(\xi + k)|^2 = 1$.

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Synthesis of two-dimensional filetr banks and wavelet functions

M. K. Tchobanou, O. V. Bolshakova

Moscow Energetics Institute (Technical University), Russia
tmk@vvk2.mpei.ac.ru

Синтез двумерных банков фильтров и вейвлет функций

Предлагается метод построения ортогональных банков фильтров (БФ) для 4-хканальных многоскоростных систем. Подход состоит в рассмотрении каскадных форм, гарантирующих структурную ортогональность, линейность фазы (ЛФ) и гладкость характеристик. Каскадная форма позволяет уменьшить число неизвестных. Их выбор осуществляется при оптимизации нелинейной задачи. Такой каскад не полный, т.е. не любой ортогональный БФ с ЛФ может быть описан этой формой.

Полифазная матрица будет равна [1]

$$\mathcal{H}(z_1, z_2) = \mathbf{R_1 W P} \prod_{\mathbf{i=2}}^{\mathbf{K}} \mathbf{D}(z_1, z_2)\mathbf{P W R_i W P},$$

где

$$R_i = \begin{bmatrix} \cos\alpha_i & -\sin\alpha_i & 0 & 0 \\ \sin\alpha_i & \cos\alpha_i & 0 & 0 \\ 0 & 0 & \cos\beta_i & -\sin\beta_i \\ 0 & 0 & \sin\beta_i & \cos\beta_i \end{bmatrix}, \; W = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix},$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \; D(z_1, z_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & z_1 & 0 & 0 \\ 0 & 0 & z_2 & 0 \\ 0 & 0 & 0 & z_1 z_2 \end{bmatrix}.$$

Тогда $\mathbf{H}_j(z_1, z_2) = \mathcal{H}_{j,0}(z_1^2, z_2^2) + \mathcal{H}_{j,1}(z_1^2, z_2^2)z_1 + \mathcal{H}_{j,2}(z_1^2, z_2^2)z_2 + \mathcal{H}_{i,3}(z_1^2, z_2^2)z_1 z_2, \; j = 0 \ldots 3.$

Фильтры $\mathbf{H}_0$ и $\mathbf{H}_1$ являются симметричными, а $\mathbf{H}_2$ и $\mathbf{H}_3$ — антисимметричными. Каждый фильтр имеет ЛФ и импульсную характеристику (маску) квадратной формы размером $2K \times 2K$. Полученный БФ ортогонален. Для заданного K нужно выбрать углы $\alpha_1 ... \alpha_K$ и $\beta_1 ... \beta_K$. Необходимо получить максимально плоские фильтры из условия равенства нулю производных $\dfrac{\partial^{k_1+k_2}\mathbf{H}_j}{\partial z_1^{k1} z_2^{k2}} = 0$ в точках $(1,1), (1,-1), (-1,1)$ или $(-1,-1)$. Построены соответствующие вейвлет функции [2].

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Problems of generating wavelets similar to test functions

Maria I. Toma, Cristian I. Toma

Politehnica University, Bucharest, Romania

cgtoma@physics1.physics.pub.ro

As it is known, in averaging procedures the user is interested in the mean value of the received signal over a certain time interval. Usually this operation is performed by an integration of the signal on this time interval (considered to be constant) the result of the integration being proportional to the mean value of the signal. However, such structures are very sensitive to random variations of the integration period (generated by switching phenomena at the end of the integration). For this reason, multiplication of the recived signal with a test-function (a function which differs from zero only on this time interval, with continuous derivatives of any order on the whole real axis) is recommended. But such a function is similar to a wavelet function used for multiplying a received signal. This paper presents some invariance properties of differential equations, which can be used for generating such wavelets, consudered as "practical" test-function on a certain time interval, this time interval, and it presents also the properties of second order oscillating systems (considered as generating such wavelets) in filtering and sampling procedures.

Positive definite finite splines and approximation with linear combinations of their shifts

R. M. Trigub

Donetsk National University, Ukraine
postmaster@ok.donbass.com

Положительно определённые финитные сплайны и приближение линейными комбинациями их сдвигов

Продолжается изучение A-сплайнов, введенных автором ранее. Получена общая теорема о порядке приближения функций и их производных линейными комбинациями сдвигов A-сплайна и его производных, соответственно.

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The spline foundation of wavelet theory

Michael Unser

Swiss Federal Institute of Technology, Lausanne, Switzerland

michael.unser@epfl.ch

Recently, we came up with two interesting generalizations of polynomial splines by extending the degree of the generating functions to both real and complex exponents. While these may qualify as exotic constructions at first sight, we show here that both types of splines (fractional and complex) play a truly fundamental role in wavelet theory and that they lead to a better understanding of what wavelets really are.

To this end, we first revisit wavelet theory starting from the representation of a scaling function as the convolution of a B-spline (the regular part of it) and a distribution (the irregular or residual part). This formulation leads to some new insights on wavelets and makes it possible to re-derive the main results of the classical theory — including some new extensions for fractional orders — in a self-contained, accessible fashion. In particular, we prove that the B-spline component is entirely responsible for five key wavelet properties: order of approximation, reproduction of polynomials, vanishing moments, multi-scale differentiation, and smoothness (regularity) of the basis functions.

Second, we show that any scaling function can be expanded as a sum of harmonic splines (a particular subset of the splines with complex exponents); these play essentially the same role here as the Fourier exponentials do for periodic signals. This harmonic expansion provides an explicit time domain representation of scaling functions and wavelets; it also explains their fractal nature. Remarkably, truncating the expansion preserves the essential multiresolution property (two scale relation). Keeping the first term alone yields a fractional spline approximation that captures most of the important wavelet features; e.g., its general shape and smoothness.

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Precise Jackson-type inequalities for linear methods of spline approximation of differentiable functions

O. L. Vinogradov†, V. V. Zhuk‡

St. Petersburg State University, Russia

†*olvin@math.spbu.ru*, ‡*zhuk@math.spbu.ru*

Точные неравенства типа Джексона для линейных методов приближения дифференцируемых функций сплайнами[16]

Для широкого класса линейных методов приближения периодических функций установлены оценки их отклонений, точные в смысле постоянных в равномерной и интегральной метрике, посредством линейных комбинаций модулей непрерывности различных порядков. В частности, установлена следующая теорема для приближения сплайнами.

[16]Работа выполнена при поддержке гранта РФФИ № 02-01-01112 и МАС № 03-01-06465.

Пусть $n, r, m, \mu \in \mathbf{N}$, $\mu \geq r + m$, r нечетно, $1 \leq p \leq \infty$, $f \in W_p^{(r)}$. Тогда

$$\|f - X_{n,r,\mu}(f)\|_p \leq \left(\frac{\pi}{n}\right)^r \left\{ \frac{A_{r,0}}{2}\omega_1\left(f^{(r)}, \frac{\pi}{n}\right)_p + \right.$$

$$\left. + \sum_{\nu=1}^{m-1} A_{r,\nu} \left\| \delta^{\nu}_{\frac{\pi}{n}}(f^{(r)}) \right\|_p + \left(\frac{\mathcal{K}_r}{\pi^r} - \sum_{\nu=0}^{m-1} 2^{\nu} A_{r,\nu}\right) 2^{-m} \left\| \delta^{m}_{\frac{\pi}{n}}(f^{(r)}) \right\|_p \right\},$$

причем при $p = 1, \infty$ константы на множестве $W_p^{(r)}$ не могут быть уменьшены. Здесь $\mathcal{K}_r = \frac{4}{\pi} \sum\limits_{l=0}^{\infty} \frac{(-1)^{l(r+1)}}{(2l+1)^{r+1}}$, $A_{r,\nu}$ — некоторые явно построенные константы, $\delta_h^{\nu}(f)$ — разность порядка ν функции f с шагом h, $\omega_1(f, h)_p$ — модуль непрерывности функции f, а $X_{n,r,\mu}$ — явно построенные линейные операторы со значениями в пространстве периодических сплайнов порядка μ минимального дефекта с $2n$ равноотстоящими узлами. Правая часть неравенства принимает наименьшее значение по m при наибольшем допустимом m, т.е. при $m = \mu - r$.

Следствием приведенной теоремы при $m = 1$ является точное неравенство типа Джексона

$$\|f - X_{n,r,\mu}(f)\|_p \leq \frac{\mathcal{K}_r}{2n^r}\,\omega_1\left(f^{(r)}, \frac{\pi}{n}\right)_p \qquad (\mu \geq r + 1).$$

On convergence of derivatives of odd-degree spline interpolation and de Boor's problem

Yu. S. Volkov

Sobolev Institute of Mathematics, Novosibirsk, Russia

volkov@math.nsc.ru

Let $\Delta = \{a = x_0 < x_1 < \ldots < x_N = b\}$ be a partition of the interval $[a, b]$ and $S = S_{2k-1}(f)$ be a spline of the odd degree

$2k-1$ which interpolates f on Δ with some boundary condition (for uniqueness), $S \in C^{2k-2}[a,b]$.

A general problem of odd-degree spline interpolation is to study the convergence of derivatives $S_{2k-1}^{(m)}(f)$ to $f^{(m)}$ in sup-norm independently of Δ. If $f \in C^m[a,b]$ then the equivalent form of this problem is to bound $P^{(m)} = \sup\limits_{f \in C^m} \dfrac{||S^{(m)}(f)||_\infty}{||f^{(m)}||_\infty}$ independently of the mesh.

C. de Boor [1] established that $P^{(m)}$ cannot be bounded for $m < k-1$, the only candidates are $P^{(k-1)}$ and $P^{(k)}$. The problem of boundedness $P^{(k)}$ independently of Δ was solved by A. Yu. Shadrin [2], but the question of bounding $P^{(k-1)}$ remained open.

In this talk we prove the mesh-independent boundedness $P^{(k-1)}$, i.e. $||S_{2k-1}^{(k-1)} - f^{(k-1)}||_\infty \to 0$ $(||\Delta|| \to 0)$ for arbitrary sequence Δ and any $f \in C^{k-1}[a,b]$. Moreover, there are directly connected the convergence conditions for $||S_{2k-1}^{(m)} - f^{(m)}||_\infty$ with $f \in C^m[a,b]$ and for $||S_{2k-1}^{(2k-1-m)} - f^{(2k-1-m)}||_\infty$ with $f \in C^{2k-1-m}[a,b]$.

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Projections and nonlinear approximation in the space $BV(\mathbb{R}^d)$

P. Wojtaszczyk

Warsaw University, Poland

wojtaszczyk@mimuw.edu.pl

In this talk we want to present results about non-linear approximation in the L_p norm $p = d/(d-1)$ of functions of bounded variation on $\mathbb{R}^d$ with $d > 1$ by polynomials in the Haar system. The exponent p is the natural exponent as it is the correct exponent in the Sobolev inequality. The space of functions of bounded variation turned out to be quite useful in geometric measure theory, calculus of variation and differential equations. More recently (but really only in the case $d = 2$) this space was used in the image processing.

The approximation schemes we are discussing in this paper are mostly related to Haar tresholding and m–term approximation. Those problems for $d = 2$ were studied in detail in the paper by A. Cohen, R. DeVore, P. Petrushev and H. Xu, *Nonlinear approximation and the space* $BV(\mathbb{R}^2)$, Amer. J. Math. 121 (1999), pp. 587–628. The study of the case $d > 2$ is justified by simple mathematical curiosity but also by the fact that minimization of various functionals related to variation (also for dimension > 2) plays an important role in problems of differential equations. The ingredients in our analysis are

1. boundedness in $\mathrm{BV}(\mathbb{R}^d)$ of certain class of averaging operators.

2. nonlinear approximation in L_p. For this we use (variants of) recent results on greedy approximation with respect to the Haar system

This talk is based on my paper *Projections and non-linear approximation in the space* $BV(\mathbb{R}^d)$ which is to appear in Proc. London Math. Soc.

On some properties of Buhmann functions

V. P. Zastavnyi
Donetsk National University, Ukraine
matan@matfak.dongu.donetsk.ua

О некоторых свойствах функций Бухмана

Пусть $\delta, \mu, \nu \in \mathbb{C}_+ := \{z \in \mathbb{C} : \operatorname{Re} z > 0\}$. В 1998 г. Buhmann ввёл следующие чётные функции: $\varphi_{\delta,\mu,\nu,\alpha}(x) := \int_x^1 (1 - s^\delta)^{\mu-1} \times \times (s^2 - x^2)^{\nu-1} s^{\alpha-2\nu+1}\, ds$ при $|x| < 1$ и $\varphi_{\delta,\mu,\nu,\alpha}(x) := 0$ при $|x| \geq 1$. В частных случаях получаются известные функции: $\mu\delta\varphi_{\delta,\mu,1,\delta}(x) = (1 - |x|^\delta)_+^\mu$ и $\varphi_{1,\mu,\nu,2\nu-1} \equiv h_{\mu,\nu} \equiv \frac{2^{\nu-1}\Gamma(\nu)}{\mu}\psi_{\mu,\nu-1}$, где функции $h_{\mu,\nu}$ рассмотрены в работах Заставного и Тригуба, а функции $\psi_{\mu,\nu-1}$ при $\mu, \nu - 1 \in \mathbb{N}$ ввёл Wendland. В теореме 1 найден точный показатель гладкости функций $\varphi_{1,\mu,\nu,\alpha}$ (подобные утверждения доказаны и при других δ).

Теорема 1.
1) *Пусть* $\delta, \mu, \nu \in \mathbb{C}_+$. *Тогда* $\varphi_{\delta,\mu,\nu,\alpha} \in C(\mathbb{R}) \iff \alpha, \mu+\nu-1 \in \mathbb{C}_+$.
2) *Пусть* $\mu, \nu, \alpha, \mu + \nu - 1 \in \mathbb{C}_+$ *и*

$$p = \max\{k \in \mathbb{Z} : k < \min\{\operatorname{Re}(\mu + \nu - 1), \operatorname{Re}\alpha\}\}.$$

Тогда $\varphi_{1,\mu,\nu,\alpha}(||\cdot||_2) \in C^p(\mathbb{R}^m)$. *Если* $\operatorname{Re}(\mu + \nu - 1) \leq \operatorname{Re}\alpha$, *то* $\varphi_{1,\mu,\nu,\alpha}(||\cdot||_2) \notin C^{p+1}(\mathbb{R}^m)$. *Если* $\operatorname{Re}(\mu+\nu-1) > \operatorname{Re}\alpha$ *и выполнено одно из трёх условий: i)* $\alpha = 2\nu - 1 \in \mathbb{C}_+$*; ii)* $\nu = 1$, $\frac{\alpha}{2} \notin \mathbb{N}$*; iii)* $\nu, \frac{\alpha-2\nu+3}{2} \in \mathbb{N}$, *то* $\varphi_{1,\mu,\nu,\alpha}(||\cdot||_2) \notin C^{p+1}(\mathbb{R}^m)$.
3) *Пусть* $\mu, \nu, \alpha, \mu + \nu - 1 \in \mathbb{C}_+$, $\alpha = 2(\nu - 1 + s), s \in \mathbb{N}$ *и* $q = \max\{k \in \mathbb{Z} : k < \min\{\operatorname{Re}(\mu + \nu - 1), \operatorname{Re}(\alpha + 1)\}\}$. *Тогда* $\varphi_{1,\mu,\nu,\alpha}(||\cdot||_2) \in C^q(\mathbb{R}^m)$. *Если выполнено одно из двух условий: i)* $\operatorname{Re}(\mu+\nu-1) \leq \operatorname{Re}(\alpha+1)$*; ii)* $\operatorname{Re}(\mu+\nu-1) > \operatorname{Re}(\alpha+1), \nu \in \mathbb{N}$, *то* $\varphi_{1,\mu,\nu,\alpha}(||\cdot||_2) \notin C^{q+1}(\mathbb{R}^m)$.

Для функций h, заданных на $(0, +\infty)$, при $m \in \mathbb{C}_+$ и $t > 0$ определим преобразование

$$\mathfrak{F}_m(h)(t) := t^{1-\frac{m}{2}} \int_0^{+\infty} h(u) u^{\frac{m}{2}} J_{\frac{m}{2}-1}(tu)\, du,$$

где J_λ — функция Бесселя первого рода. При $m \in \mathbb{N}$ преобразование $\mathfrak{F}_m$ связано с преобразованием Фурье радиальных функций: $\int_{\mathbb{R}^m} h(||u||_2)e^{-iux}\,du = (2\pi)^{\frac{m}{2}}\mathfrak{F}_m(h)(||x||_2)$, $x \in \mathbb{R}^m$. При $\delta, \mu, \nu + \frac{1}{2}, \alpha + 1 \in \mathbb{C}_+$ и $t > 0$ положим

$$I_{\delta,\mu,\nu,\alpha}(t) := t^{-\alpha-1-\delta(\mu-1)} \int_0^t (t^\delta - u^\delta)^{\mu-1} u^{\alpha-\nu+\frac{1}{2}} J_{\nu-\frac{1}{2}}(u)\,du.$$

Очевидно функция $I_{\delta,1,\nu,\alpha}$ не зависит от δ и $I_{\delta,\mu+1,\nu-1,2\nu-2} \equiv \delta\mu I_{\delta,\mu,\nu,2\nu-2+\delta}$, если дополнительно $\nu - \frac{1}{2} \in \mathbb{C}_+$.

Теорема 2. *Пусть $\delta, \mu, \nu, m, \alpha + m \in \mathbb{C}_+$. Тогда*

$$\mathfrak{F}_m(\varphi_{\delta,\mu,\nu,\alpha})(t) = 2^{\nu-1}\Gamma(\nu) I_{\delta,\mu,\frac{m-1}{2}+\nu,m-1+\alpha}(t).$$

Если ещё $m - 1 + 2\nu \in \mathbb{C}_+$, то

$$\mathfrak{F}_m(\varphi_{\delta,\mu,\nu,\alpha})(t) = \frac{2^{\frac{1-m}{2}}\Gamma(\nu)}{\Gamma(\frac{m-1}{2}+\nu)} \mathfrak{F}_1(\varphi_{\delta,\mu,\frac{m-1}{2}+\nu,m-1+\alpha})(t), \quad t > 0.$$

Теорема 3. *Пусть $\delta, \mu, \nu + \frac{1}{2}, \alpha + 1 > 0$ и $I_{\delta,\mu,\nu,\alpha}(t) \geq 0$ при $t > 0$. Тогда:*
1) $I_{\delta,\mu+\tau,\nu,\alpha}(t) > 0$, $I_{\delta,\mu,\nu+\tau,\alpha}(t) > 0$ и $I_{\delta,\mu+\varepsilon,\nu,\alpha-\delta\varepsilon}(t) > 0$ при всех $\tau > 0$, $\varepsilon \in (0, \frac{\alpha+1}{\delta})$ и $t > 0$.
2) Если дополнительно $\alpha + 1 \geq \delta$, то $I_{\delta-\varepsilon,\mu,\nu,\alpha-\varepsilon}(t) > 0$ при всех $\varepsilon \in (0, \delta)$ и $t > 0$.
3) Если дополнительно $\mu > 1$, то $I_{\varepsilon,\mu,\nu,\alpha}(t) > 0$ при всех $\varepsilon \in (0, \delta)$ и $t > 0$.
4) Если дополнительно $\mu \leq 1$, то $I_{\varepsilon,\tau,\nu,\alpha}(t) > 0$ при всех $\varepsilon > 0, \tau > 1$ и $t > 0$.

Утверждение 1) в теореме 3 при $\delta = 1$ и $\delta = 2$ доказал Gasper. Утверждение 2) при $\nu = 1, \alpha = \delta$ доказал Kuttner, а при $2\nu - 1 \in \mathbb{N}$, $\alpha = 2\nu - 2 + \delta$ Голубов. Из теоремы 3 получаются результаты работ Misiewicz и Richards, Buhmann.

Теорема 4. *Неравенство $I_{\delta,\mu,\nu,\alpha}(t) \geq 0$ не может выполняться при всех $t > 0$ в следующих случаях: i) $\delta, \mu, \nu, \alpha > 0$, $\mu + \nu - 1 \leq 0$; ii) $\delta = 1$, $0 < \mu < \nu$, $\alpha \geq 2\nu - 1$; iii) $\delta = 1$,*

$0 < \mu = \nu < 1$, $\alpha \geq 2\nu - 1$; *iv)* $\alpha > 2\nu \geq 0$, $\mu > 0$, $\delta > 0$; *v)* $\alpha \geq 2\nu > -1$, $\mu > 0$, $\delta \geq 2$; *vi)* $\alpha \geq 2\nu > -1$, $0 < \mu \leq 1$, $\delta > 0$; *vii)* $\nu \geq 1$, $\delta > 0$, $\alpha \geq 2\nu + \delta - 2$, $0 < \mu < \delta + \nu - 1$.

В теореме 4 случай iii) при $\alpha = 2\nu - 1$ рассмотрел Moak. Случаи v) и vii) при $\alpha = 2\nu + \delta - 2$ и $2\nu - 1 \in \mathbb{N}$ рассмотрел Голубов (при $\nu = 1$ Kuttner).

Data interpolation by splines on triangulations

Frank Zeilfelder

University of Mannheim, Germany

zeilfeld@euklid.math.uni-mannheim.de

The literature on local Hermite interpolation by smooth splines on triangulations is vast. On the other hand, since recently no results of this type were known for interpolation involving only data which is called Lagrange interpolation. We develop the first local Lagrange interpolation methods for smooth splines on triangulations and triangulated quadrangulations. The methods are based on a suitable coloring of the triangles and quadrilaterals, respectively. The Lagrange interpolating splines yield optimal approximation order and can be computed with linear complexity. The interpolation methods are general in the sense that they are the basis for the construction of spline surfaces in 3D space and for pure approximation approaches extending local approximations to bivariate splines. Our recent investigations show that for splines with higher smoothness the coloring can be combined with an appropriate combinatorical approach in order to organize interpolation points such that they are local. We give numerical tests with synthetic as well as real world data involving up to 10^6 interpolation points which demonstrate the efficiency of the methods. The results were obtained by joint work with Günther Nürnberger (University of Mannheim, Mannheim, Germany), Vera Rayevskaya, and Larry L. Schumaker (Vanderbilt University, Nashville, U.S.A.).

Spline subdivision schemes

Valery A. Zheludev[†], Amir Z. Averbuch[‡]

Tel Aviv University, Israel

[†]*zhel@post.tau.ac.il*, [‡]*amir1@post.tau.ac.il*

A univariate stationary uniform subdivision scheme (SS) S_a consists in the following: A function f^j defined on the grid $\mathbf{G^j} = \{k/2^j\}_{k\in\mathbb{Z}}$: $f^j(k/2^j) = f^j_k$, is extended onto the grid $\mathbf{G^{j+1}}$ by filtering the array $\{f^j_k\}$: $f^{j+1}_k = \sum_{l\in\mathbb{Z}} a_{k-2l} f^j_l$. This is one refinement step. The filter $\mathbf{a} = \{a_k\}$ is called the refinement mask of the SS S_a. If $a_0 = 1,\ a_{2k} = 0\ \forall k \neq 0$ then the SS is interpolatory (ISS). The z-transform $a(z)$ of the mask is called the symbol of the SS S_a. In our approach we start with construction of a polynomial spline of deficiency 1 or a discrete spline approximating data given on the initial grid. One step of subdivision consists in calculation of values of the spline on a refined grid. Then the procedure is iterated. This approach enables to devise a diverse library of subdivision schemes and to flexibly apply these schemes to CAGD problems. This diversity stems from the multitude of available splines and the variety of ways of refinement. The polynomial interpolatory and smoothing splines are feasible for this purpose as well as quasi-interpolatory splines.

Interpolatory polynomial splines of even degrees constructed on uniform grid possess a property which is valuable for subdivision: In the midpoints between the points of interpolation these splines restore polynomials of the degree which is higher than their own degree. This property is called the *super-convergence*. Due to this property the limit functions of the corresponding ISS turn out to be more regular than the splines themselves. Actually, these limit functions form a new class of functions which deserves a thorough investigation. Also, the approximation order of the ISS is higher than that of the splines.

A seeming drawback of usage of the interpolatory splines is that it requires convolution the data with the infinite mask. But, due to rational structure of the symbols this obstacle could be circumvented by employing the recursive filtering. As a result , the computational cost of implementation of such schemes is even lower then the cost of implementation of schemes with the finite masks which have comparable properties. If the initial data on the grid $\{k\}$ is the Kroneker delta then the limit function $\varphi(x)$ of a SS is called the basic limit function (BLF) of the SS. Then the limit function $f^\infty(x)$ which emerges from the initial data $\{f_k^0\}$ is the sum $f^\infty(x) = \sum_{k\in\mathbb{Z}} f_k^0 \varphi(x-k)$. If the mask is finite than BLF is compactly supported. We establish conditions for BLF to be a continuous function and to decay exponentially as $x \to \infty$. Under these conditions the SS converges on any initial date of power growth.

This approach can be extended to the cases when the initial data are given on non-uniformly spaced grids in one and several dimensions.

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